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TECHNICAL MEMORANDUM NO. 21

A LINEAR CLOSED LOOP SYSTEM ANALYSIS PROCEDURE USING LINE PRINTER PLOTS OF CHARACTERISTIC EQUATION ROOT LOCI

by

Harold H. Burke
Robert L. Payne, Jr.

November 1968

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U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
ARMY MATERIEL SYSTEMS ANALYSIS AGENCY
ABERDEEN PROVING GROUND, MARYLAND

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Weapon Systems Division

RDT&E Project No. 1T523801A098

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ABSTRACT

An existing program that determines the root locus of n'th order polynominals has been modified to provide plots of these loci in the complex frequency plane using a standard line printer. A methodology that combines the computational capabilities of this root locus program with a variable scale graphical display of selectable regions of the complex frequency plane is presented. A listing of the Fortran IV source deck of the modified program and two examples are included.

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I. INTRODUCTION

The stability of linear closed loop systems is effectively determined through the use of root locus methods[1]. For all but the most trivial cases, the polynomial representing the system's characteristic equation is high order. Manual calculation of the roots of this polynomial is laborious and time consuming. A Fortran IV program [2] has been developed to expedite this process. The interpretation of a tabular display even when available is at best cumbersome. The purpose of this memo is to describe the use of a modified version of this basic program which provides selectable scaled graphical displays of the root loci as part of the tabular output.

Main features of the modified program are:

1. Programmed in Fortran IV. No machine oriented or object language.
2. No complex arithmetic.
3. No special graphical plotting equipment necessary.
4. Order of polynomial may be up to 100.
5. Number of variations of coefficient may be up to 100.

Main features of the graphical display are:

1. Log plot of third and fourth quadrants of complex frequency plane from 0 to 10,000 radians/second.
2. Linear plots of selected regions of the third and fourth quadrants of the complex frequency plane with arbitrary scales.

II. NATURE OF THE PROBLEM

Regardless of the complexity of a closed loop system its transfer function can be reduced to the equivalent form shown in Figure I. For multiple loop systems, the G's and H's are readily expressed as sums of products of polynomials which are identified with individual elements making up the complete system.

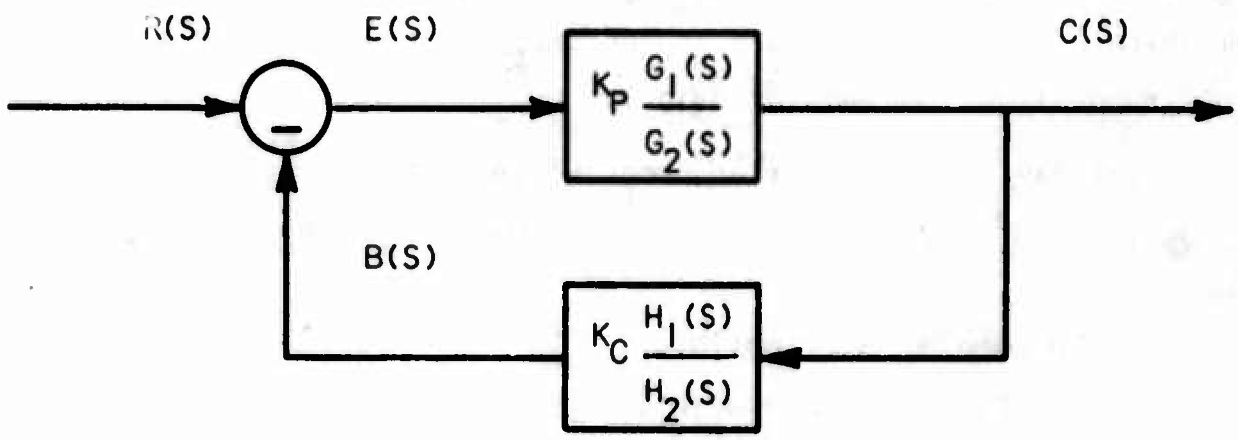

Figure I. Linear Closed Loop System

where:

	R(S)	=	System Input
	C(S)	=	System Output
	B(S)	=	System Feedback
	E(S)	=	System Error
	K_p	=	Process Gain
	K_c	=	Controller Gain

The fractions $G_1(S)/G_2(S)$ and $H_1(S)/H_2(S)$ are equivalent transfer functions of the system and are represented by ratios of polynomials which upon expansion can be put in the form

$$\frac{S^n + a_1 S^{n-1} + a_2 S^{n-2} + \dots a_n}{S^{m+n} + b_1 S^{m+n-1} + b_2 S^{m+n-2} + b_{m+n}} = \frac{\sum_{u=o}^{n} a_u S^{n-u}}{\sum_{i=o}^{m+n} b_i S^{m+n-i}} \qquad (1)$$

or in factored form

$$\frac{\prod_{u=1}^{n} (S + Z_u)}{\prod_{i=1}^{m+n} (S + p_i)} \qquad (2)$$

where the a's and b's are real, the poles and zeros ($-p_i$ and $-Z_u$) can be either real or complex in conjugate pairs and $m \geq 1$.

The linear closed loop system transfer function is

$$\frac{C(S)}{R(S)} = \frac{K_p G_1(S) H_2(S)}{K_C K_p G_1(S) H_1(S) + G_2(S) H_2(S)} \qquad (3)$$

The linear closed loop system's characteristic equation which determines the roots of the transfer function's denominator is

$$K_C K_p G_1(S) H_1(S) + G_2(S) H_2(S) = 0. \qquad (4)$$

If we let $K_C K_p = + K^*$ (5)

$G_1(S) H_1(S) = A(S)$

$G_2(S) H_2(S) = B(S)$

then equation (4) can be written as

$$K^*A(S) + B(S) = 0 \tag{6}$$

and the linear closed loop system transfer function is

$$\frac{C(S)}{R(S)} = \frac{K_p G_1(S)\, H_2(S)}{K^*A(S) + B(S)} \tag{7}$$

The transfer function for a unit step in R(S) becomes

$$C(S) = \frac{K_p \prod_{u=1}^{n} (S + Z_u)}{S \prod_{i=1}^{m+n} (S + p_i)} \tag{8}$$

and the time response of the output, c(t) to the unit step is given by

$$c(t) = L^{-1}[C(S)] \tag{9}$$

where $L^{-1}(\ )$ indicates the inverse Laplace transform

Expressing equation (8) in partial fraction form we have

$$C(S) = \frac{K_o}{S} + \sum_{i=1}^{m+n} \frac{K_i}{(S+p_i)} \tag{10}$$

where K_o and K_i are the residues at the respective poles of C(S).

Specifically

$$K_o = \frac{K_p \prod_{u=1}^{n} (Z_u)}{\prod_{i=1}^{m+n} (p_i)} \tag{11}$$

$$K_\ell = \left. \frac{[K_p \prod_{u=1}^{n} (S + Z_u)]}{[S \prod_{\substack{i=1 \\ i\neq\ell}}^{m+n} (S + p_i)]} \right|_{S=-p_\ell} \tag{12}$$

The inverse transform of (10)is therefore

$$c(t) = K_o + \sum_{i=1}^{m+n} K_i e^{p_i t} \tag{13}$$

If a pole p_ℓ is complex, its corresponding residue, K_ℓ, is also complex. In this case there also exists a term of the form

$K_{\bar{\ell}}\, e^{p_{\bar{\ell}} t}$ where the bar denotes complex conjugate.

We then have

$$(K_\ell\, e^{p_\ell t} + \overline{K}_\ell\, e^{\overline{p}_\ell t}) = 2|K_\ell| e^{\sigma_\ell t} \cos(w_\ell t + \phi_\ell) \tag{14}$$

where:

$$p_\ell = \sigma_\ell + jw_\ell \tag{15}$$

$$\overline{p}_\ell = \sigma_\ell - jw_\ell$$

$$\phi_\ell = \tan^{-1} \frac{w_\ell}{\sigma_\ell}$$

Thus if j poles are real and m+n-j poles are complex we arrive at the complete time solution for c(t)

$$c(t) = K_o + \sum_{i=1}^{j} K_i e^{p_i t} + \sum_{\ell=j+1}^{K} 2|K_\ell|\, e^{\sigma_\ell t} \cos\,(w_\ell t + \phi_\ell) \tag{16}$$

where

$$K = \frac{m + n - j}{2}$$

As the value of K* is varied, the roots of the system's characteristic equation change. Since $-\frac{1}{p_i}$ is the time constant for a single pole and $-\sigma_\ell$ is the damping coefficient associated with the frequency w_ℓ, we can see that the locus of p_i and p_ℓ in the complex plane, as K* varies, gives an indication of the stability of the system.

Although the stability of the system for a unit step function is generally adequate the response of the system F(t) to an arbitrary input R(t) can be found by the **Duhamel** integral

$$F(t) = \int_0^t R(u)\ c(t-u)\ du \qquad (17)$$

where c(t) is of the form given in equations (13) and (15). A locus of roots as K^* varies from K^*_{min} to K^*_{max} is called the root locus of the system. From (6) it is seen that when K_c is not present (no closed loop system), $K^*=0$ and the roots of the characteristic equation are the roots of B(S). When K_c is present (closed loop system) and the value of K_c approaches infinity, K^* also approaches infinity. The roots of the characteristic equation are the roots of A(S).

III. GRAPHICAL METHOD

The root loci of the system are the trajectories of the roots of equation (6) in the complex plane as K^* varies from 0 to ∞. It will be recalled that since the coefficients of the characteristic equation are real the complex roots appear in conjugate pairs and the loci are symmetric about the σ axis. Hence investigation of the half plane reveals the nature of the entire set of loci. In our plots of the loci, we choose to display only the lower half on the complex frequency plane, i.e., the third and fourth quadrants which make up the left and right hand quadrants respectively. We include two options (1) a log plot and (2) an expanded scale linear plot.

In the log plot separate plots are made of the left and right hand quadrants. Each is partitioned into decades ranging from 0.01 rad/sec to 10,000 rad/sec. Figures 3 and 4 show the actual log plots for example 1, to be described later. Figures 5 and 6 show two actual linear expands for example 1, to be described later.

Detailed plotting accuracy is of no concern on the log plots. Selection of the proper scale will provide the desired accuracy on the linear expand plots. The left hand quadrant coordinates are minus sigma and j-omega. The right hand quadrant coordinates are plus sigma and j-omega. The dimensions of the coordinates in the complex frequency plane are radians/second.

Three different symbols are shown in the complex frequency plane log plot. The definition of these are

- * Roots of B(S)
- 0 Roots of A(S)
- ● Roots of K*A(S) + B(S)

Dependent on the proximity of any of the above roots priority is given to plotting first the roots A(S) then the roots of B(S) and finally the roots of K*A(S) + B(S). For all real roots which are located on the plus sigma or minus sigma axis, the roots of A(S) appear on the sigma axis. Any other root that may occur within the stepping increment of a root of A(S) i.e., other roots of A(S), B(S) or K*A(S) + B(S) will appear to the right. The same priority of graphical printout is maintained for the complex roots.

Rewriting (6) gives

$$K*A(S) + B(S) = 0 \qquad (6)$$

Dividing (6) by B(S) and rearranging gives

$$1-\frac{[-K*A(S)]}{B(S)}$$

where $-K*\frac{A(S)}{B(S)}$ = open loop transfer function of equivalent system $[TF]_{oL}$

The algebraic sign of the ratio of the lowest powered terms of open loop transfer function, $\frac{A(S)}{B(S)}$ will determine the regions on the $\pm$ sigma axis where the roots of $K*\frac{A(S)}{B(S)}$ are located. For a positive A(S) /B(S) the roots of 1-[TF] open loop = 0 are located on the zero

angle locus [3] and extend from + infinity on the positive sigma axis to the largest positive root of A(S) or B(S). From this most positive real root the locus of roots are located in the even numbered intervals between the roots of A(S) and B(S) to minus infinity on the sigma axis. For a A(S)/B(S) the roots of K*A(S)+B(S) = 0 are located on the 180° angle locus and extend from the largest positive root of A(S) or B(S) on the Sigma axis leftward to the adjacent root and in the odd numbered intervals between the roots of A(S) and B(S) to minus infinity on the sigma axis.

Inspection of the log plot shown in Figures 3 and 4 demonstrate the real locus for a positive $\frac{A(S)}{B(S)}$, while Figures 5 and 6 demonstrate the real locus for a negative $\frac{A(S)}{B(S)}$. When applying this method a pencilled line parallel to the sigma axis to designate these closed loop root trajectories on the real axis is suggested.

The precise location of these roots is given in the numerically tabulated output. A detailed discussion of the complete input and output will be later. Mention is made here only to indicate that close continuity exists between the numerical and graphical output.

Regions of interest on the complex plane are obvious from the log plots. In order to study segments of the root loci more closely a linear expand plot of the complex frequency plane is used. Similar to the log plot, only one quadrant of the right and left hand planes are shown. The scaling of the linear expand plots is completely arbitrary. The regions of interest are determined from inspection of the log plots. As many regions as desired can be expanded. Figures 5 and 6 are one such linear expand on a specific region of the complex frequency plane.

IV. DATA FORMATS

A. Polynomial Multiplication and Root Locus Method

1. General Description.

a. Computes the numerator polynomial A, where A is the sum of the products of several sets of polynomials. Each set contains a variable number of polynomials and a variable number of sets form the sum.

b. Computes the denominator polynomial B, where B is the sum of the products of several sets of polynomials.

c. Computes the roots of A.

d. Computes the roots of B.

e. Computes the polynomial K * A + B, where K*varies from K initial to K terminate in increments of ΔK, or particular values of K*may be chosen.

f. Computes the roots of the K * A + B polynomials.

2. Input.

	Description	Columns	Data
a.	Identification	1 - 80	Identification of run
b.	Control card (Integers)	1 - 10	0 for ΔK N for number of input particular values of K
		11 - 20	Number of polynomial groups to be added in A(s)
		21 - 30	Number of polynomial groups to be added in B(s)
		31 - 40	Problem number
c.	If Δk is used		
		1 - 10	K initial
		11 - 20	ΔK

2. <u>Input</u>. (Cont.)

Description	Columns	Data
c1. If Δk is used		
	21 - 30	K terminate
c2. If specific values of K are used,		
	1 - 10	K_1
	11 - 20	K_2
	21 - 30	K_3
	o	
	o	
	o	
	o	
	o	K_N
	(seven values per card)	
d. Numerator group count (Integers)		
	1 - 10	Number of polynomials in group 1 of numerator
	11 - 20	Number of polynomials in group 2 of numerator
	o	
	o	
	o	
	Etc.	
	(seven values per card)	

2. Input (Cont.)

Description	Columns	Data
e. Denominator group count (integers)		
	1 - 10	Number of polynomials in group 1 of denominator
	11 - 20	Number of polynomials in group 2 of denominator
	0	
	0	
	0	
	Etc. (seven values per card)	

f. Polynomials. Seven values per card, with each polynomial starting on a new card. The first value of each polynomial is an interger, with the coefficients in ascending order in floating point:

Columns	Data
1 - 10	Degree + 1 of the polynomial
11 - 20	Constant term
21 - 30	Coefficient of X
31 - 40	Coefficient of X^2
Etc.	

Load in polynomials in the order in which they appear in the fraction, with the numerator polynomials first.

3. Output.

The output consists of:

a. Value of K* initial, increment K*, value of K* terminate.

b. Number of polynomial groups added in A(S).

c. Number of polynomial groups added in B(S).

d. Number of polynomials in each group of A(S).

e. Number of polynomials in each group of B(S).

f. Coefficients of polynomials in each group of A(S) and B(S).

3. Output (Cont.)

g. The degree, coefficients and roots of A.

h. The degree, coefficients and roots of B.

i. The degree, coefficients, roots, value of K* for each K* A + B polynomial.

4. Special Considerations.

a. The maximum order of A or B is 100.

b. The maximum number of specific values of K is 100.

B. Root Locus Plot Subroutine

1. General Description. This is a subroutine designed to work with the basic polynomial multiplication and root locus program. This subroutine takes the roots of K*A + B calculated by the polynomial root locus program and plots them on the complex frequency plane with the following options:

Log plots of the third and fourth quadrants of the complex frequency plane from 0 to 10,000 rad/sec. Linear expand plots of the third and fourth quadrants of the complex frequency plane with selectable scale. The linear expand plots supplement the log plot. The region covered by any one linear expand plot can be designated by the analyst through the use of data cards. The sigma dimensions are determined by choosing a certain point in either the third or fourth quadrant and picking a percentage of this point to be the distance represented on the sigma axis. (See linear expand output of example). The J-omega dimensions are determined by choosing a certain point on the J-omega axis and picking a percentage of this point to be the distance represented on the J-omega axis.

2. Input.

Description	Columns	Data	
Control Card Used to call the log plot subroutine	1	0 (zero)	integer
	11	0 (zero)	integer
	21	0 (zero)	integer
	31 - 35	10000(ten thousand)	integer
Control Cards Used to call the expand option			
Number of Expands Desired	1 - 10		integer
For each linear Expand, data appears on one card	1 - 10	J-omega specific	Real
	11 - 20	J-omega percent	Real
	21 - 30	Sigma specific	Real
	31 - 40	Sigma percent	Real

Note: There must be as many data cards as there are number of expands desired.

3. Output.

Depending on the options specified the output consists of:

a. A list of all of the roots plotted on the log plot.

b. Two log plots of the roots calculated by the polynomial multiplication and root locus program.

c. The specified number of linear expand plots of selectable regions in the complex frequency plane.

V. EXAMPLES

Two examples of the use of the techniques described above will be considered. The first is the realization of a typical control system and will be discussed in some detail. The second is included simply to indicate the results for a fairly large order system.

Example 1.

A typical control system is shown in Figure 2.

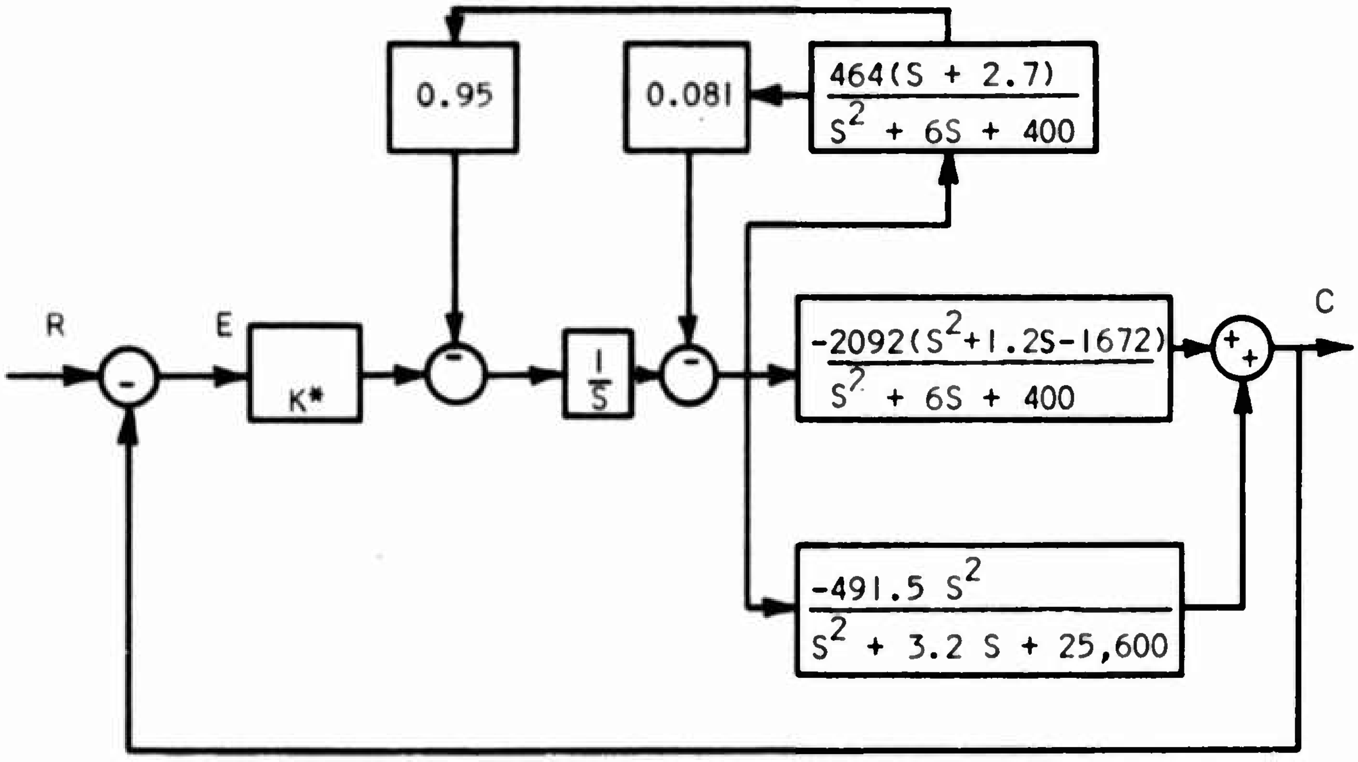

Figure 2. Linear Closed Loop Control System Block Diagram

The equivalent form of Figure 2 is shown in Figure 1 where $G_1(S)\ H_1(S) = A(S)$ and $G_2(S)\ H_2(S) = B(S)$ and $K^* = K_p K_c$.

The characteristic equation for the system shown in Figure 2 is

$$-K^*[2092(S^2+1.2S-1672)(S^2+3.2S+25{,}600) + 491.5S^2(S^2+6S+400)]$$

$$+(0.081)(464)(S+2.7)\ (S^2+3.2S + 25{,}600)S$$

$$+(0.95)\ (464)\ (S + 2.7)\ (S^2 + 3.2S + 25{,}600)$$

$$+S(S^2 + 6S + 400)\ (S^2 + 3.2S + 25{,}600) = 0$$

Table 1 shows the input card format for this characteristic equation. Lines 1 through 20 apply to the polynomial multiplication and root locus method and lines 21 through 28 apply to the root locus plot subroutine.

Table 1. Input for Example 1

0	2	3	331
0.0	0.01	0.3	
3	2		
4	4	3	
3	25600.	3.2	1.
3	-1672.	1.2	1.
1	-2092.		
3	0.	0.	-491.5
3	400.	6.	1.
1	.081		
2	0.	464.	
2	2.7	1.	
3	25600.	3.2	1.
1	.95		
1	464.		
2	2.7	1.	
3	25600.	3.2	1.
2	0.	1.	
3	400.	6.	1.
3	25600.	3.2	1.

POLYNOMIAL MULTIPLICATION AND ROOT LOCUS PROGRAM

0	0	0	10000
6			
100.	100.	-50.	100.
100.	100.	100.	100.
50.	100.	-50.	100.
50.	100.	50.	100.
150.	20.	-5.	100.
150.	20.	5.	100.

ROOT LOCUS PLOT SUBROUTINE

The tabulated numerical output is shown in Tables 2 through 5. Table 2 mirrors the number of inputs values of K*, (0= ΔK increment, N= number of particular Input values of K*), the number polynomial groups added in A(S) and B(S), the problem number, K* initial, increment K*, K* terminate, the number of polynomial in each group of A(S) and B(S), and the coefficients of each of these polynomials proceeding from the left most term in the numerator to the final term in the denominator. Each polynomial group appears on a separate sheet. Table 3 defines the equivalent open loop A(S) and B(S) in single polynomial and root form. Table 4 defines K* A(S) + B(S) in polynomial and root form for values of K* between K* initial and K* terminate. Table 5 is a tabulation of K* A(S) + B(S) roots which are to be plotted.

Figures 3 and 4 are the log plots for Example I. The net sign for the lowest order coefficients is positive as seen in Table 4, hence the zero degree locus is plotted. The roots of A(S) and B(S) are

A(S)
39.918653
-41.162138
$-1.7304538 \pm j\ 145.22688$

B(S)
-1.3440477
$-1.6000000 \pm j\ 159.99200$
$-21.119976 \pm j\ 20.963084$

The locus of the K* A(S) + B(S) roots on the sigma axis as K* varys from 0 to 0.3 in increments of 0.010 may be determined by inspection. It appears to the right of 39.918653 and between -1.3440477 and -41.162138. The complex locus has two branches. One branch comes from the complex pair at $-21.119976 \pm j\ 20.963084$ and the other from the complex pair at $-1.6000000 \pm j\ 159.99200$.

For K* gains between 0 and 0.3 these two branches lie in the third and fourth quadrants. Inspection of this log plot in conjunction with the open loop roots of A(S) and B(S) given in Table 4 provides insight

into the regions of the complex plane where an expanded scale plot of the root locus is required. The region bounded by $-100. < \sigma < 200$ and $0 < w < 200$ is shown in Figures 5 and 6. The locus between sigma equal to -41.162138 and -1.3440477 is shown in detail. The locus between sigma equal to $-21.119976 \pm j\, 20.963084$ and the positive sigma axis roots + 39.918653 and plus infinity is shown in detail. The locus in the 100 - 1000 rad/second j-omega decade is shown in a compressed form in this plot. Figures 7 and 8 are another set of expanded scale plots of the region bounded by $-100 < \sigma < 100$ and $0 < w 100$. This scale provides more sensitivity to that portion of the locus which progresses from -21.119976 $\pm$ 20.963084 to the positive sigma axis. The locus in the 100-1000 rad/sec j-omega decade is not shown. Figures 9 and 10 are expanded scale plots in the region bounded by $-10 < \sigma < 10$ and $20 < w < 180$. The roots of A(S) and B(S) are in the third (left hand) quadrant while the roots of K*A(S) + B(S) are in the fourth (right hand) quadrant. The migration of these roots as K* is varied is an indication of the requirement for this expanded scale plot of the complex frequency plane.

Example 2

Table 6 is the tabulated numerical output for a more complex multiloop system whose characteristic equation is of 24'th order Figures 11 and 12 are log plots. The off axis printing on the sigma axis is shown. The net sign of the lowest order coefficients of A(S)/B(S) is negative, hence the 180° locus is plotted. Inspection shows that the locus is between +0.0040482270 and -0.046149160 and then between each alternate root of A(S) or B(S) on the sigma axis. Inspection of Figures 11 and 12 indicates that several regions of the complex frequency plane are of interest. One such region is the region bounded by $-16.0 < \sigma < 16.0$ and $25. < w < 175.$ which is shown in Figures 13 and 14.

TABLE 2

Mirror of Input, Example 1

```
POLYNOMIAL MULTIPLICATION AND ROOT LOCUS                                   PROBLEM NO.  331

   DELTAK=          0     POLY.ADDED IN A(S)=        2     POLY.ADDED IN B(S)=        3     PROB.NO.=        331

K-INITIAL=          .0000000000      INCREMENT K=        .0100000000     K-TERMINATE=        .3000000000
   NUMBER OF POLY. IN GROUP  1 OF NUMERATOR=        3
   NUMBER OF POLY. IN GROUP  2 OF NUMERATOR=        2
   NUMBER OF POLY. IN GROUP  1 OF DENOMINATOR=        4
   NUMBER OF POLY. IN GROUP  2 OF DENOMINATOR=        4
   NUMBER OF POLY. IN GROUP  3 OF DENOMINATOR=        3
   C(   1)=    25600.0000000000
   C(   2)=        3.2000000000
   C(   3)=        1.0000000000
   C(   1)=    -1672.0000000000
   C(   2)=        1.2000000000
   C(   3)=        1.0000000000
   C(   1)=    -2092.0000000000
```

TABLE 2

Mirror of Input, Example I. (Contd)

```
C(   1)=        .0000000000
C(   2)=        .0000000000
C(   3)=   -491.5000000000
C(   1)=    400.0000000000
C(   2)=      6.0000000000
C(   3)=      1.0000000000
```

TABLE 2

Mirror of Input, Example I. (Contd)

```
C(   1)=          .0810000000
C(   1)=          .0000000000
C(   2)=       464.0000000000
C(   1)=         2.7000000000
C(   2)=         1.0000000000
C(   1)=     25600.0000000000
C(   2)=         3.2000000000
C(   3)=         1.0000000000
```

TABLE 2

Mirror of Input, Example I. (Contd)

```
C(   1)=        .9500000000
C(   1)=     464.0000000000
C(   1)=       2.7000000000
C(   2)=       1.0000000000
C(   1)=   25600.0000000000
C(   2)=       3.2000000000
C(   3)=       1.0000000000
```

TABLE 2

Mirror of Input, Example I. (Contd)

```
C(   1)=         .0000000000
C(   2)=        1.0000000000
C(   1)=      400.0000000000
C(   2)=        6.0000000000
C(   3)=        1.0000000000
C(   1)=    25600.0000000000
C(   2)=        3.2000000000
C(   3)=        1.0000000000
```

TABLE 3

Equivalent System Open Loop Polynomials and Roots, Example I.

```
COEFFICIENTS ARE GIVEN IN ASCENDING ORDER


THE COEFFICIENTS OF POLYNOMIAL A (ORDER =   4)
 8.9544294E 10      -5.3073203E 07      -5.0262009E 07      -1.2153800E 04      -2.5835000E 03

 THE ROOTS OF A
 3.9918653E 01   +I  0.0000000E 00      -4.1162138E 01   +I  0.0000000E 00      -1.7304538E 00   +I -1.4522688E 02
-1.7304538E 00   +I  1.4522688E 02


THE COEFFICIENTS OF POLYNOMIAL B (ORDER =   5)
 3.0468096E 07       2.4126095E 07       1.1199558E 06       2.6681746E 04       4.6784000E 01       1.0000000E 00

 THE ROOTS OF B
-1.3440477E 00   +I  0.0000000E 00      -2.1119976E 01   +I  2.0963084E 01      -2.1119976E 01   +I -2.0963084E 01
-1.6000000E 00   +I  1.5999200E 02      -1.6000000E 00   +I -1.5999200E 02
```

TABLE 4

Closed Loop System Polynomials and Roots, Example I.

```
POLYNOMIAL MULTIPLICATION AND ROOT LOCUS                                                              PROBLEM NO.  331


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =    5)  K =      0.0000000E 00
 3.0468096E 07        2.4126095E 07        1.1199558E 06        2.6681746E 04        4.6784000E 01        1.0000000E 00

  ROOTS OF K*A + B
-1.3440477E 00   + I  0.0000000E 00       -2.1119976E 01   + I  2.0963084E 01       -2.1119976E 01   + I -2.0963084E 01
-1.6000000E 00   + I  1.5999200E 02       -1.6000000E 00   + I -1.5999200E 02


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =    5)  K =      1.0000000E-02
 9.2591104E 08        2.3595363E 07        6.1733575E 05        2.6560208E 04        2.0949000E 01        1.0000000E 00

  ROOTS OF K*A + B
 4.0494673E 00   + I  3.3975966E 01        4.0494673E 00   + I -3.3975966E 01       -3.0798403E 01   + I  0.0000000E 00
 8.7523407E-01   + I  1.6024308E 02        8.7523407E-01   + I -1.6024308E 02


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =    5)  K =      2.0000000E-02
 1.8213540E 09        2.3064631E 07        1.1471566E 05        2.6438670E 04       -4.8860000E 00        1.0000000E 00

  ROOTS OF K*A + B
-3.4675444E 01   + I  0.0000000E 00        1.6364826E 01   + I  4.2304285E 01        1.6364826E 01   + I -4.2304285E 01
 3.4158963E 00   + I  1.5974276E 02        3.4158963E 00   + I -1.5974276E 02


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =    5)  K =      3.0000000E-02
 2.7167969E 09        2.2533898E 07       -3.8790443E 05        2.6317132E 04       -3.0721000E 01        1.0000000E 00

  ROOTS OF K*A + B
-3.6373738E 01   + I  0.0000000E 00        2.7830727E 01   + I -4.6851801E 01        2.7830727E 01   + I  4.6851801E 01
 5.7166419E 00   + I -1.5848945E 02        5.7166419E 00   + I  1.5848945E 02


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =    5)  K =      4.0000000E-02
 3.6122399E 09        2.2003166E 07       -8.9052453E 05        2.6195594E 04       -5.6556000E 01        1.0000000E 00

  ROOTS OF K*A + B
-3.7352153E 01   + I  0.0000000E 00        3.9472012E 01   + I  4.8733260E 01        3.9472012E 01   + I -4.3733260E 01
 7.4820644E 00   + I  1.5663009E 02        7.4820644E 00   + I -1.5663009E 02
```

TABLE 4

Closed Loop System Polynomials and Roots, Example I. (Contd)

```
POLYNOMIAL MULTIPLICATION AND ROOT LOCUS                                                PROBLEM NO.  331


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     5.0000000E-02
 4.5076828E 09      2.1472434E 07      -1.3931446E 06      2.6074056E 04      -8.2391000E 01      1.0000000E 00

   ROOTS OF K*A + B
-3.7993535E 01  + I  0.0000000E 00      5.1675266E 01  + I  4.7830316E 01      5.1675266E 01  + I -4.7830316E 01
 8.5170010E 00  + I -1.5445648E 02      8.5170010E 00  + I  1.5445648E 02


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     6.0000000E-02
 5.4031258E 09      2.0941702E 07      -1.8957647E 06      2.5952518E 04      -1.0822600E 02      1.0000000E 00

   ROOTS OF K*A + B
-3.8448009E 01  + I  0.0000000E 00      6.4519180E 01  + I  4.3289137E 01      6.4519180E 01  + I -4.3289137E 01
 8.8178250E 00  + I -1.5232117E 02      8.8178250E 00  + I  1.5232117E 02


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     7.0000000E-02
 6.2985687E 09      2.0410970E 07      -2.3983848E 06      2.5830980E 04      -1.3406100E 02      1.0000000E 00

   ROOTS OF K*A + B
-3.8787478E 01  + I  0.0000000E 00      7.7857764E 01  + I  3.2945263E 01      7.7857764E 01  + I -3.2945263E 01
 8.5664744E 00  + I -1.5048875E 02      8.5664744E 00  + I  1.5048875E 02


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     8.0000000E-02
 7.1940116E 09      1.9880238E 07      -2.9010049E 06      2.5709442E 04      -1.5989600E 02      1.0000000E 00

   ROOTS OF K*A + B
-3.9050949E 01  + I  0.0000000E 00      1.0137499E 02  + I  0.0000000E 00      8.1558877E 01  + I  0.0000000E 00
 8.0065400E 00  + I  1.4905381E 02      8.0065400E 00  + I -1.4905381E 02


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     9.0000000E-02
 8.0894546E 09      1.9349506E 07      -3.4036250E 06      2.5587904E 04      -1.8573100E 02      1.0000000E 00

   ROOTS OF K*A + B
-3.9261499E 01  + I  0.0000000E 00      6.4244210E 01  + I  0.0000000E 00      7.3285547E 00  + I -1.4798418E 02
 7.3285547E 00  + I  1.4798418E 02      1.4609118E 02  + I  0.0000000E 00
```

TABLE 4

Closed Loop System Polynomials and Roots, Example I. (Contd)

```
POLYNOMIAL MULTIPLICATION AND ROOT LOCUS                                                PROBLEM NO.  331


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =      1.0000000E-01
 8.9848975E 09       1.8818774E 07       -3.9062451E 06       2.5466366E 04       -2.1156600E 02       1.0000000E 00

   ROOTS OF K*A + B
-3.9433684E 01   + I   0.0000000E 00       5.8581243E 01   + I   0.0000000E 00       6.6421054E 00   + I -1.4720164E 02
 6.6421054E 00   + I   1.4720164E 02       1.7913423E 02   + I   0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =      1.1000000E-01
 9.8803405E 09       1.8288042E 07       -4.4088652E 06       2.5344828E 04       -2.3740100E 02       1.0000000E 00

   ROOTS OF K*A + B
-3.9577150E 01   + I   0.0000000E 00       5.5277448E 01   + I   0.0000000E 00       5.9977661E 00   + I -1.4662976E 02
 5.9977661E 00   + I   1.4662976E 02       2.0970517E 02   + I   0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =      1.2000000E-01
 1.0775783E 10       1.7757310E 07       -4.9114853E 06       2.5223290E 04       -2.6323600E 02       1.0000000E 00

   ROOTS OF K*A + B
-3.9698552E 01   + I   0.0000000E 00       5.3041355E 01   + I   0.0000000E 00       5.4135301E 00   + I -1.4620872E 02
 5.4135301E 00   + I   1.4620872E 02       2.3906614E 02   + I   0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =      1.3000000E-01
 1.1671226E 10       1.7226578E 07       -5.4141054E 06       2.5101752E 04       -2.3907100E 02       1.0000000E 00

   ROOTS OF K*A + B
-3.9802630E 01   + I   0.0000000E 00       5.1405678E 01   + I   0.0000000E 00       4.8919037E 00   + I -1.4589537E 02
 4.8919037E 00   + I   1.4589537E 02       2.6768414E 02   + I   0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =      1.4000000E-01
 1.2566669E 10       1.6695846E 07       -5.9167255E 06       2.4980214E 04       -3.1490600E 02       1.0000000E 00

   ROOTS OF K*A + B
-3.9892853E 01   + I   0.0000000E 00       5.0148663E 01   + I   0.0000000E 00       4.4288833E 00   + I -1.4565946E 02
 4.4288833E 00   + I   1.4565946E 02       2.9579242E 02   + I   0.0000000E 00
```

TABLE 4

Closed Loop System Polynomial and Roots, Example I. (Contd.)

```
POLYNOMIAL MULTIPLICATION AND ROOT LOCUS                                          PROBLEM NO.  331


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     1.5000000E-01
1.3462112E 10       1.6165114E 07       -6.4193455E 06       2.4858676E 04     -3.4074100E 02      1.0000000E 00

  ROOTS OF K*A + B
-3.9971821E 01  + I  0.0000000E 00       4.9148532E 01  + I  0.0000000E 00      4.0182316E 00  + I -1.4547988E 02
 4.0182316E 00  + I  1.4547988E 02       3.2352783E 02  + I  0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     1.6000000E-01
1.4357555E 10       1.5634382E 07       -6.9219656E 06       2.4737138E 04     -3.6657600E 02      1.0000000E 00

  ROOTS OF K*A + B
-4.0041520E 01  + I  0.0000000E 00       4.8331837E 01  + I  0.0000000E 00      3.6533814E 00  + I -1.4534184E 02
 3.6533814E 00  + I  1.4534184E 02       3.5097892E 02  + I  0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     1.7000000E-01
1.5252998E 10       1.5103650E 07       -7.4245857E 06       2.4615600E 04     -3.9241100E 02      1.0000000E 00

  ROOTS OF K*A + B
-4.0103495E 01  + I  0.0000000E 00       4.7651245E 01  + I  0.0000000E 00      3.3282133E 00  + I -1.4523480E 02
 3.3282133E 00  + I  1.4523480E 02       3.7820682E 02  + I  0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     1.8000000E-01
1.6148441E 10       1.4572918E 07       -7.9272058E 06       2.4494062E 04     -4.1824600E 02      1.0000000E 00

  ROOTS OF K*A + B
-4.0158964E 01  + I  0.0000000E 00       4.7074705E 01  + I  0.0000000E 00      3.0373178E 00  + I -1.4515121E 02
 3.0373178E 00  + I  1.4515121E 02       4.0525562E 02  + I  0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =     1.9000000E-01
1.7043884E 10       1.4042186E 07       -8.4298259E 06       2.4372524E 04     -4.4408100E 02      1.0000000E 00

  ROOTS OF K*A + B
 4.6579653E 01  + I  0.0000000E 00      -4.0208902E 01  + I  0.0000000E 00      2.7760322E 00  + I -1.4508556E 02
 2.7760322E 00  + I  1.4508556E 02       4.3215818E 02  + I  0.0000000E 00
```

TABLE 4

Closed Loop System Polynomials and Roots, Example I (Contd.)

```
POLYNOMIAL MULTIPLICATION AND ROOT LOCUS                                         PROBLEM NO.  331


  THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.0000000E-01
1.7939327E 10       1.3511454E 07      -8.9324460E 06       2.4250986E 04      -4.6991600E 02       1.0000000E 00

 ROOTS OF K*A + B
4.6149699E 01   + I  0.0000000E 00      -4.0254097E 01  + I  0.0000000E 00       2.5403847E 00  + I -1.4503379E 02
2.5403847E 00   + I  1.4503379E 02       4.5893963E 02  + I  0.0000000E 00


  THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.1000000E-01
1.8834770E 10       1.2980722E 07      -9.4350661E 06       2.4129449E 04      -4.9575100E 02       1.0000000E 00

 ROOTS OF K*A + B
4.5772627E 01   + I  0.0000000E 00      -4.0295196E 01  + I  0.0000000E 00       2.3270093E 00  + I -1.4499285E 02
2.3270093E 00   + I  1.4499285E 02       4.8561955E 02  + I  0.0000000E 00


  THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.2000000E-01
1.9730213E 10       1.2449990E 07      -9.9376862E 06       2.4007910E 04      -5.2158600E 02       1.0000000E 00

 ROOTS OF K*A + B
4.5439133E 01   + I  0.0000000E 00      -4.0332732E 01  + I  0.0000000E 00       2.1330558E 00  + I -1.4496045E 02
2.1330558E 00   + I  1.4496045E 02       5.1221349E 02  + I  0.0000000E 00


  THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.3000000E-01
2.0625656E 10       1.1919258E 07      -1.0440306E 07       2.3886372E 04      -5.4742100E 02       1.0000000E 00

 ROOTS OF K*A + B
4.5141995E 01   + I  0.0000000E 00      -4.0367149E 01  + I  0.0000000E 00       1.9561081E 00  + I -1.4493483E 02
1.9561081E 00   + I  1.4493483E 02       5.3873394E 02  + I  0.0000000E 00


  THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.4000000E-01
2.1521099E 10       1.1388526E 07      -1.0942926E 07       2.3764834E 04      -5.7325600E 02       1.0000000E 00

 ROOTS OF K*A + B
4.4875519E 01   + I  0.0000000E 00      -4.0398821E 01  + I  0.0000000E 00       1.7941126E 00  + I -1.4491462E 02
1.7941126E 00   + I  1.4491462E 02       5.6519108E 02  + I  0.0000000E 00
```

TABLE 4

Closed Loop System Polynomials and Roots, Example I. (Contd.)

```
POLYNOMIAL 'ULTIPLICATION AND ROOT LOCUS                                                  PROBLEM NO.  331


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.5000000E-01
 2.2416542E 10      1.0857794E 07      -1.1445546E 07      2.3643296E 04      -5.9909100E 02      1.0000000E 00

   ROOTS OF K*A + B
 4.4635150E 01  + I  0.0000000E 00      -4.0428064E 01  + I  0.0000000E 00      1.6453179E 00  + I -1.4489878E 02
 1.6453179E 00  + I  1.4489878E 02       5.9159328E 02  + I  0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.6000000E-01
 2.3311985E 10      1.0327062E 07      -1.1948167E 07      2.3521758E 04      -6.2492600E 02      1.0000000E 00

   ROOTS OF K*A + B
 4.4417202E 01  + I  0.0000000E 00      -4.0455148E 01  + I  0.0000000E 00      1.5082247E 00  + I -1.4488646E 02
 1.5082247E 00  + I  1.4488646E 02       6.1794750E 02  + I  0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.7000000E-01
 2.4207428E 10      9.7963297E 06      -1.2450787E 07      2.3400220E 04      -6.5076100E 02      1.0000000E 00

   ROOTS OF K*A + B
 4.4218654E 01  + I  0.0000000E 00      -4.0480303E 01  + I  0.0000000E 00      1.3815444E 00  + I -1.4487702E 02
 1.3815444E 00  + I  1.4487702E 02       6.4425956E 02  + I  0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.8000000E-01
 2.5102871E 10      9.2655977E 06      -1.2953407E 07      2.3278682E 04      -6.7659600E 02      1.0000000E 00

   ROOTS OF K*A + B
 4.4037009E 01  + I  0.0000000E 00      -4.0503728E 01  + I  0.0000000E 00      1.2641649E 00  + I -1.4486992E 02
 1.2641649E 00  + I  1.4486992E 02       6.7053439E 02  + I  0.0000000E 00


   THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    2.9000000E-01
 2.5[illegible]98313E 10      8.7348657E 06      -1.3456027E 07      2.3157144E 04      -7.0243100E 02      1.0000000E 00

   ROOTS OF K*A + B
 4.3870184E 01  + I  0.0000000E 00      -4.0525595E 01  + I  0.0000000E 00      1.1551225E 00  + I -1.4486475E 02
 1.1551225E 00  + I  1.4486475E 02       6.9677617E 02  + I  0.0000000E 00
```

TABLE 4

Closed Loop System Polynomials and Roots, Example I. (Contd.)

```
POLYNOMIAL MULTIPLICATION AND ROOT LOCUS                                                PROBLEM NO.  331


  THE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =   5)  K =    3.0000000E-01
2.6893756E 10       8.2041336E 06      -1.3958647E 07       2.3035606E 04      -7.2826600E 02       1.0000000E 00

  ROOTS OF K*A + B
4.3716425E 01  + I  0.0000000E 00      -4.0546057E 01  + I  0.0000000E 00       1.0535790E 00  + I -1.4486115E 02
1.0535790E 00  + I  1.4486115E 02       7.2298847E 02  + I  0.0000000E 00
```

TABLE 5
Tabulation of Open and Closed System Roots, Example 1

THE FOLLOWING ROOTS ARE PLOTTED

SIGMA	J-OMEGA
39.918652815	.000000000
-41.162138411	.000000000
-1.730453835	-145.226876122
-1.344047696	.000000000
-21.119976152	-20.963084143
-1.600000000	-159.991999800
-1.344047696	.000000000
-21.119976152	-20.963084143
-1.600000000	-159.991999800
4.049467329	-33.975965742
-30.798402795	.000000000
.875234068	-160.243084341
-34.675444105	.000000000
16.364825726	-42.304285154
3.415896326	-159.742760839
-36.373738324	.000000000
27.830727273	-46.851800735
5.716641889	-158.489445208
-37.352152913	.000000000
39.472012057	-48.733259559
7.482064399	-156.630090123
-37.993534767	.000000000
51.675266342	-47.830315926
8.517001042	-154.456482021
-38.448009292	.000000000
64.519179607	-43.289137257
8.817825039	-152.321167086
-38.787477725	.000000000
77.857764477	-32.945263258
8.566474385	-150.488748933
-39.050948789	.000000000
101.374991629	.000000000
81.558877252	.000000000
8.006539954	-149.053811987
-39.261498996	.000000000
64.244210458	.000000000
7.328554678	-147.984179529
146.091179182	.000000000
-39.433683558	.000000000
58.581242789	.000000000
6.642105422	-147.201636726
179.134229926	.000000000
-39.577150069	.000000000
55.277448021	.000000000
5.997766078	-146.629758476
209.705169891	.000000000
-39.698552007	.000000000
53.041354920	.000000000
5.413530148	-146.208717669
239.066136792	.000000000
-39.802629544	.000000000
51.405678465	.000000000
4.891903727	-145.895365670
267.684143624	.000000000

TABLE 5

Tabulation of Open and Closed System Roots, Example I. (Contd)

-39.892852699	.000000000
50.148663380	.000000000
4.420883308	-145.659455309
295.792422702	.000000000
-39.971820902	.000000000
49.148532277	.000000000
4.018231617	-145.479883461
323.527825392	.000000000
-40.041520391	.000000000
48.331836564	.000000000
3.653381446	-145.341835921
350.978920935	.000000000
-40.103495241	.000000000
47.651245034	.000000000
3.328213307	-145.234798142
378.206823593	.000000000
-40.158964097	.000000000
47.074705241	.000000000
3.037317836	-145.151210253
405.255623183	.000000000
46.579652970	.000000000
-40.208901728	.000000000
2.776032179	-145.085563619
432.158184400	.000000000
46.149698852	.000000000
-40.254097202	.000000000
2.540384711	-145.033791406
458.939628928	.000000000
45.772627107	.000000000
-40.295196145	.000000000
2.327009277	-144.992853581
485.619550483	.000000000
45.439132638	.000000000
-40.332731949	.000000000
2.133055800	-144.960450973
512.213487711	.000000000
45.141994575	.000000000
-40.367149172	.000000000
1.956108118	-144.934825743
538.733938362	.000000000
44.875518566	.000000000
-40.398821306	.000000000
1.794112605	-144.914620368
565.191077530	.000000000
44.635150405	.000000000
-40.428064441	.000000000
1.645317889	-144.898776716
591.593278258	.000000000
44.417202176	.000000000
-40.455147899	.000000000
1.508224688	-144.886462947
617.947496348	.000000000
44.218654218	.000000000
-40.480302574	.000000000
1.381544378	-144.877019957
644.259559601	.000000000
44.037009376	.000000000
-40.503727559	.000000000
1.264164870	-144.869921713
670.534388442	.000000000

TABLE 6

Equivalent System Open Loop Polynomials and Roots, Example 2

COEFFICIENTS ARE GIVEN IN ASCENDING ORDER

THE COEFFICIENTS OF POLYNOMIAL A (ORDER = 20)

-4.0799305E 19	6.1680749E 21	9.5684282E 23	2.2379228E 24	7.1031974E 23	2.3741745E 22
9.5066517E 19	8.7039308E 18	2.3363110E 18	7.5304025E 16	1.9097026E 15	4.1899455E 13
5.5180490E 11	7.7883807E 09	7.1958497E 07	6.3221235E 05	4.4817804E 03	2.2388089E 01
1.2305666E-01	2.8328917E-04	1.1650003E-06			

THE ROOTS OF A

-1.0698598E-02	+I	0.0000000E 00	-5.0000000E-01	+I	0.0000000E 00	4.0482270E-03	+I	0.0000000E 00
-3.0000000E 00	+I	0.0000000E 00	-1.8661114E 01	+I	-1.3795784E 01	-1.8661114E 01	+I	1.3795784E 01
1.7611750E 01	+I	1.5133260E 01	1.7611750E 01	+I	-1.5133260E 01	-5.6548983E-01	+I	5.8604508E 01
-5.6548983E-01	+I	-5.8604508E 01	-1.1383567E 01	+I	9.9133387E 01	-1.1383567E 01	+I	-9.9133387E 01
-4.5000000E 01	+I	0.0000000E 00	8.7784732E 00	+I	-1.0034827E 02	8.7784732E 00	+I	1.0034827E 02
-3.1400000E 01	+I	-1.8535922E 02	-3.1400000E 01	+I	1.8535922E 02	-1.2000000E 02	+I	0.0000000E 00
-1.7100326E 00	+I	-1.6740767E 02	-1.7100326E 00	+I	1.6740767E 02			

THE COEFFICIENTS OF POLYNOMIAL B (ORDER = 24)

4.4350635E 18	2.7125958E 19	6.4758656E 20	1.6958813E 21	9.5767814E 20	3.3602128E 20
4.7035417E 19	3.1814755E 18	1.2920488E 17	3.9974714E 15	1.0012828E 14	1.9949174E 12
3.4265212E 10	4.7944633E 08	5.9492045E 06	5.8870857E 04	5.5411254E 02	3.8548769E 00
2.8002114E-02	1.3359248E-04	7.3606134E-07	2.2365276E-09	8.7580072E-12	1.3306326E-14
3.0508488E-17					

THE ROOTS OF B

-4.6149160E-01	+I	-2.1047476E-14	-1.2319939E-02	+I	-8.5130501E-02	-1.2319939E-02	+I	8.5130501E-02
-1.4701442E 00	+I	2.1868848E 00	-1.4701442E 00	+I	-2.1868848E 00	-1.7866417E 01	+I	-3.0829112E-14
-1.2500000E 01	+I	7.9241888E-14	-4.2279259E-01	+I	-5.2021278E 01	-4.2279259E-01	+I	5.2021278E 01
-4.3533614E 01	+I	-4.4791865E 01	-4.3533614E 01	+I	4.4791865E 01	-3.0696042E 01	+I	0.0000000E 00
-6.2707067E-01	+I	-6.0964654E 01	-6.2707067E-01	+I	6.0964654E 01	-1.5717536E 00	+I	-1.1108758E 02
-1.5717536E 00	+I	1.1108758E 02	-2.7401987E 00	+I	-1.3136153E 02	-2.7401987E 00	+I	1.3136153E 02
-1.9168424E 00	+I	-1.5927933E 02	-1.9168424E 00	+I	1.5927933E 02	-3.1369099E 01	+I	-1.8532335E 02
-3.1369099E 01	+I	1.8532335E 02	-1.0365000E 02	+I	-3.2947940E 02	-1.0365000E 02	+I	3.2947940E 02

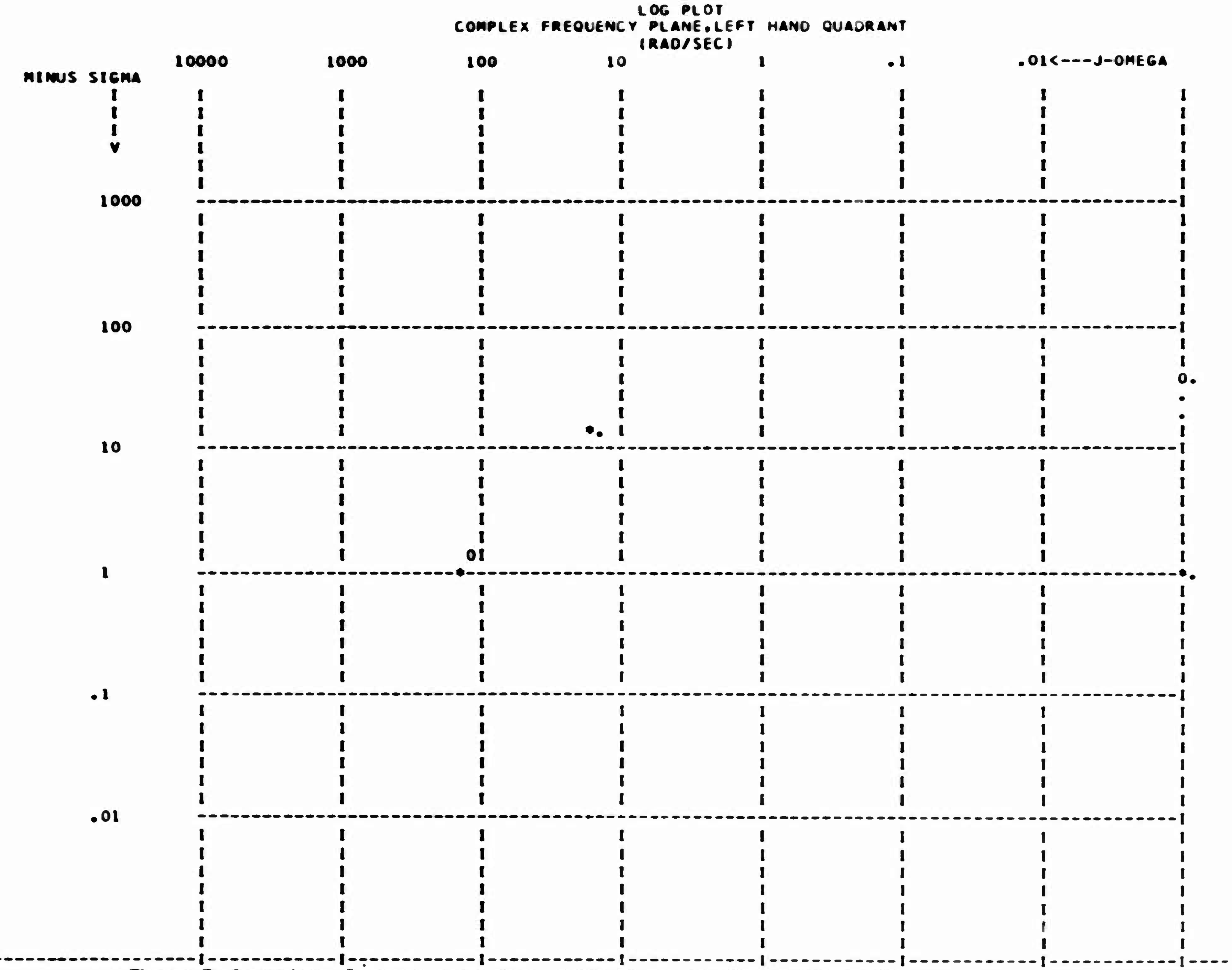

Figure 3. Graphical Display, Log Plot, Left Hand Quadrant, Example 1.

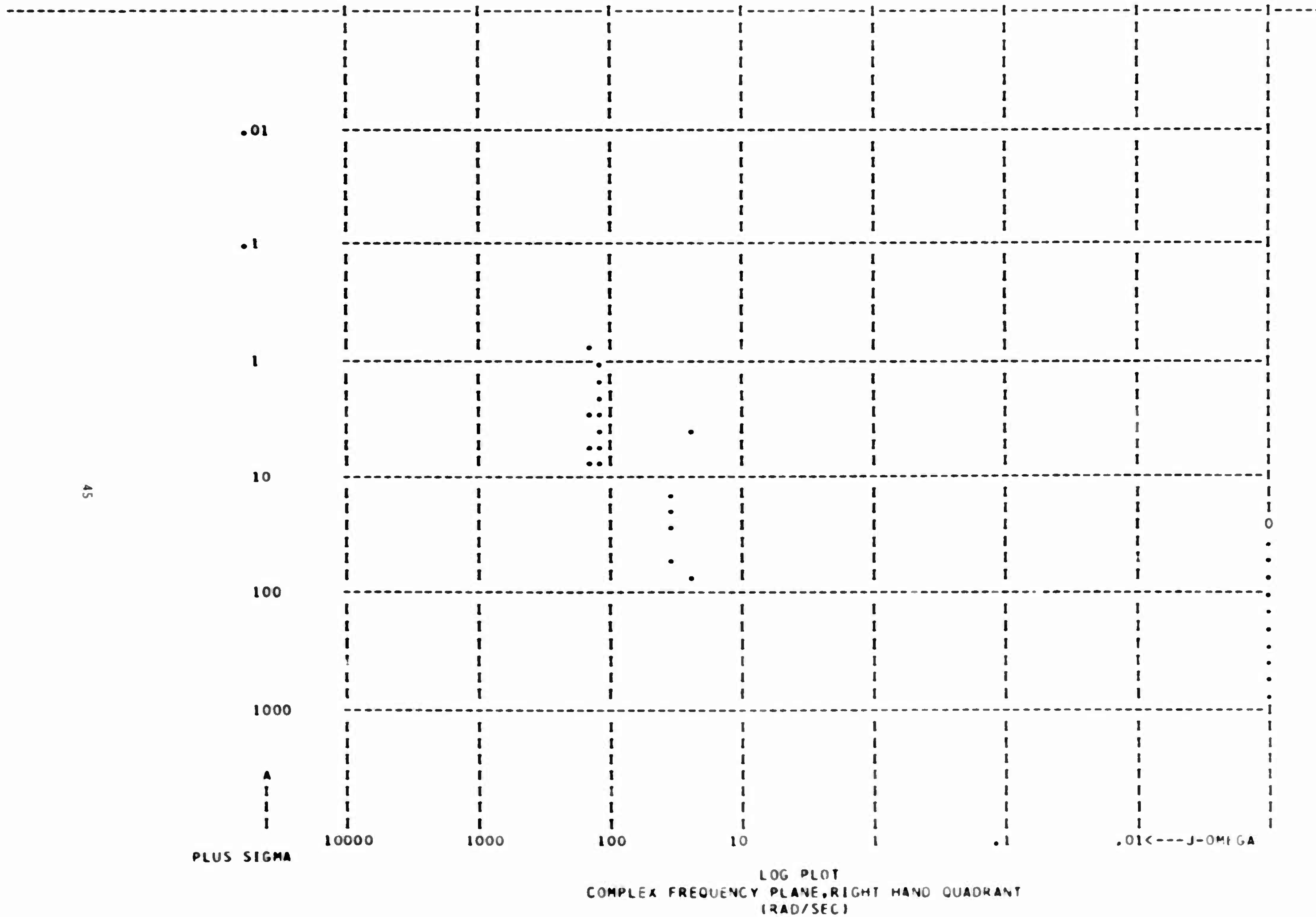

Figure 4. Graphical Display, Log Plot, Right Hand Quadrant, Example I

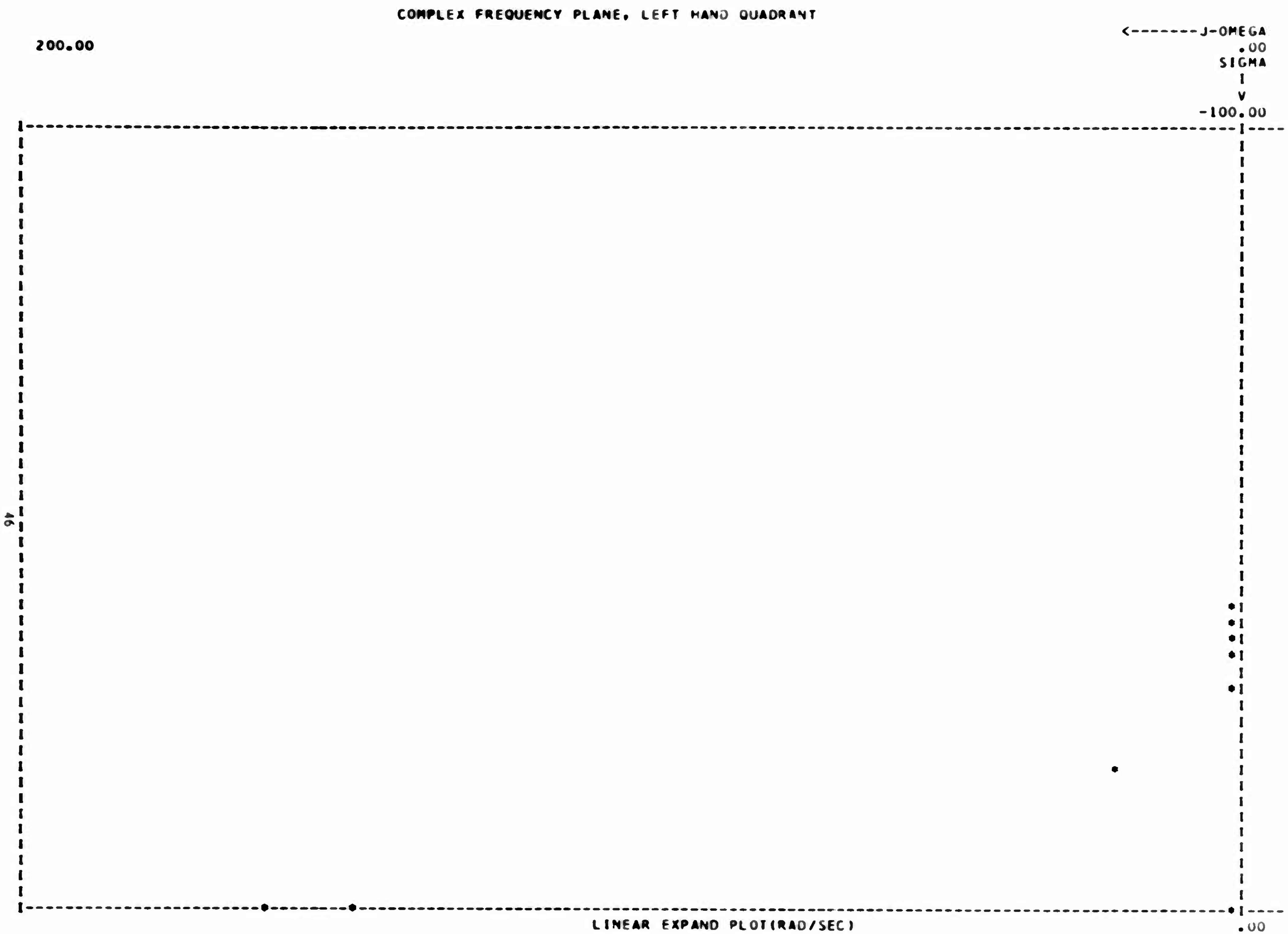

Figure 5. Graphical Display, Linear Expand Plot, Left Hand Quadrant, Example 1.

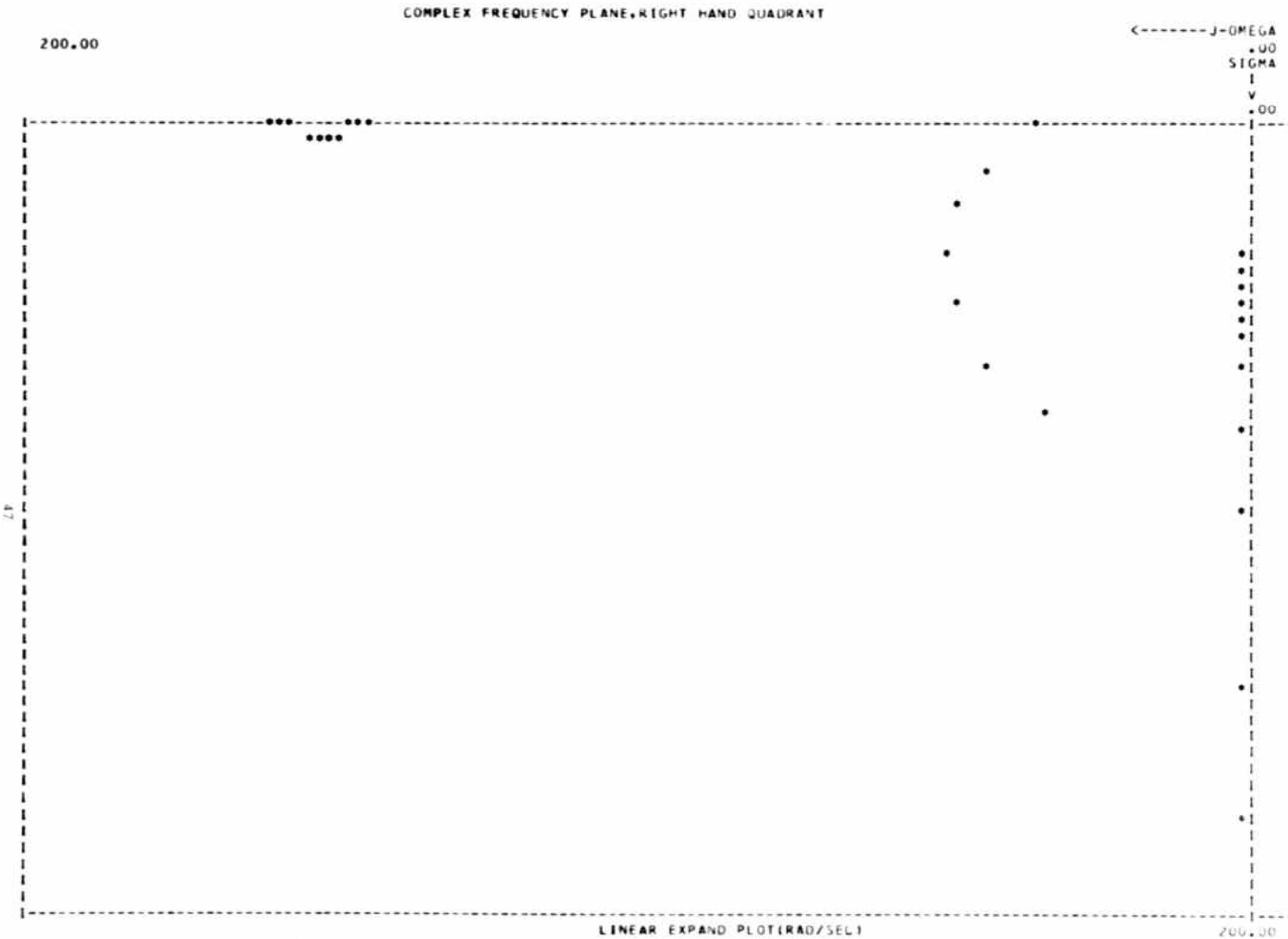

Figure 6. Graphical Display, Linear Expand Plot, Right Hand Quadrant, Example 1.

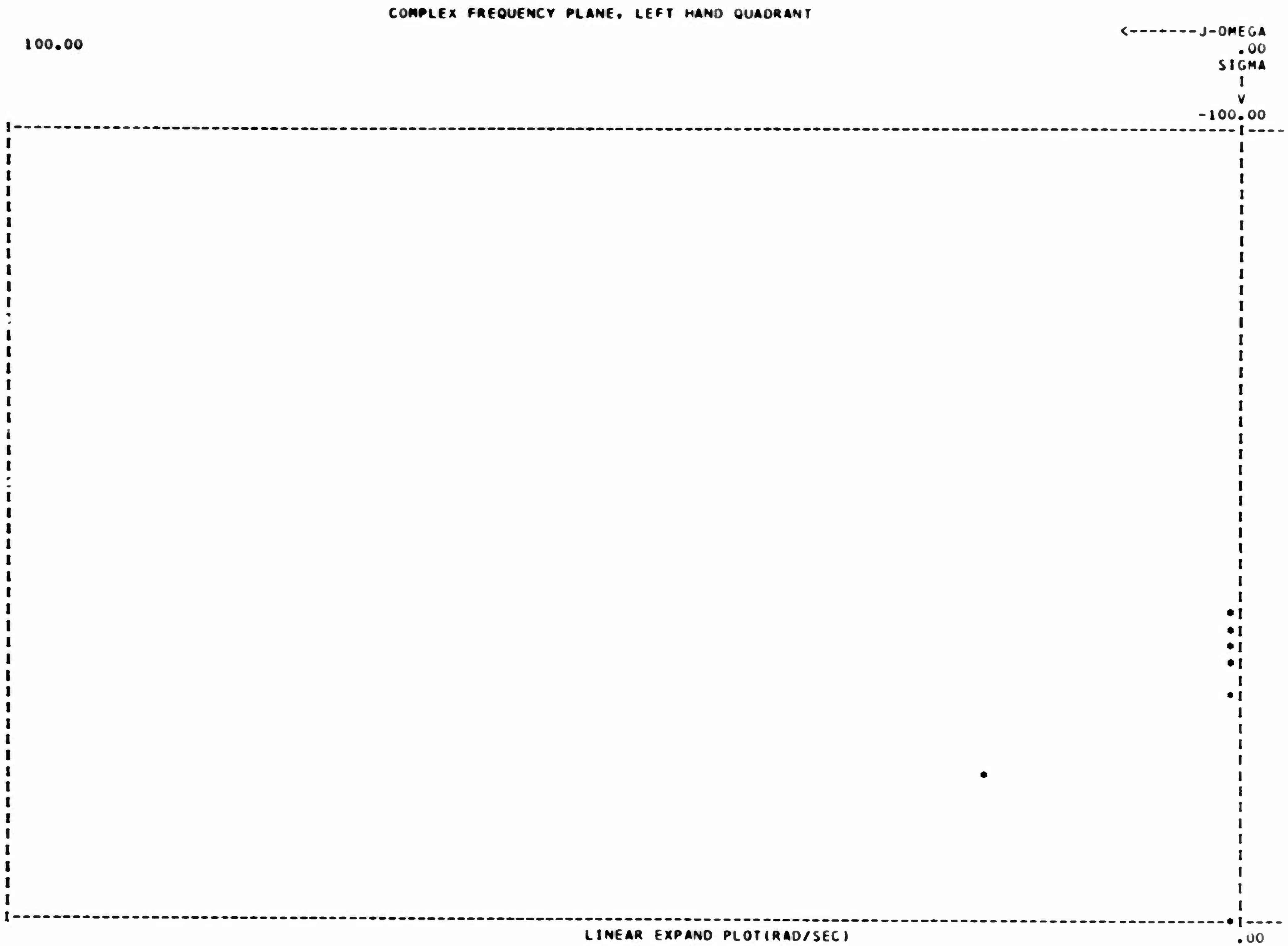

Figure 7. Graphical Display, Linear Expand Plot, Left Hand Quadrant, Example I.

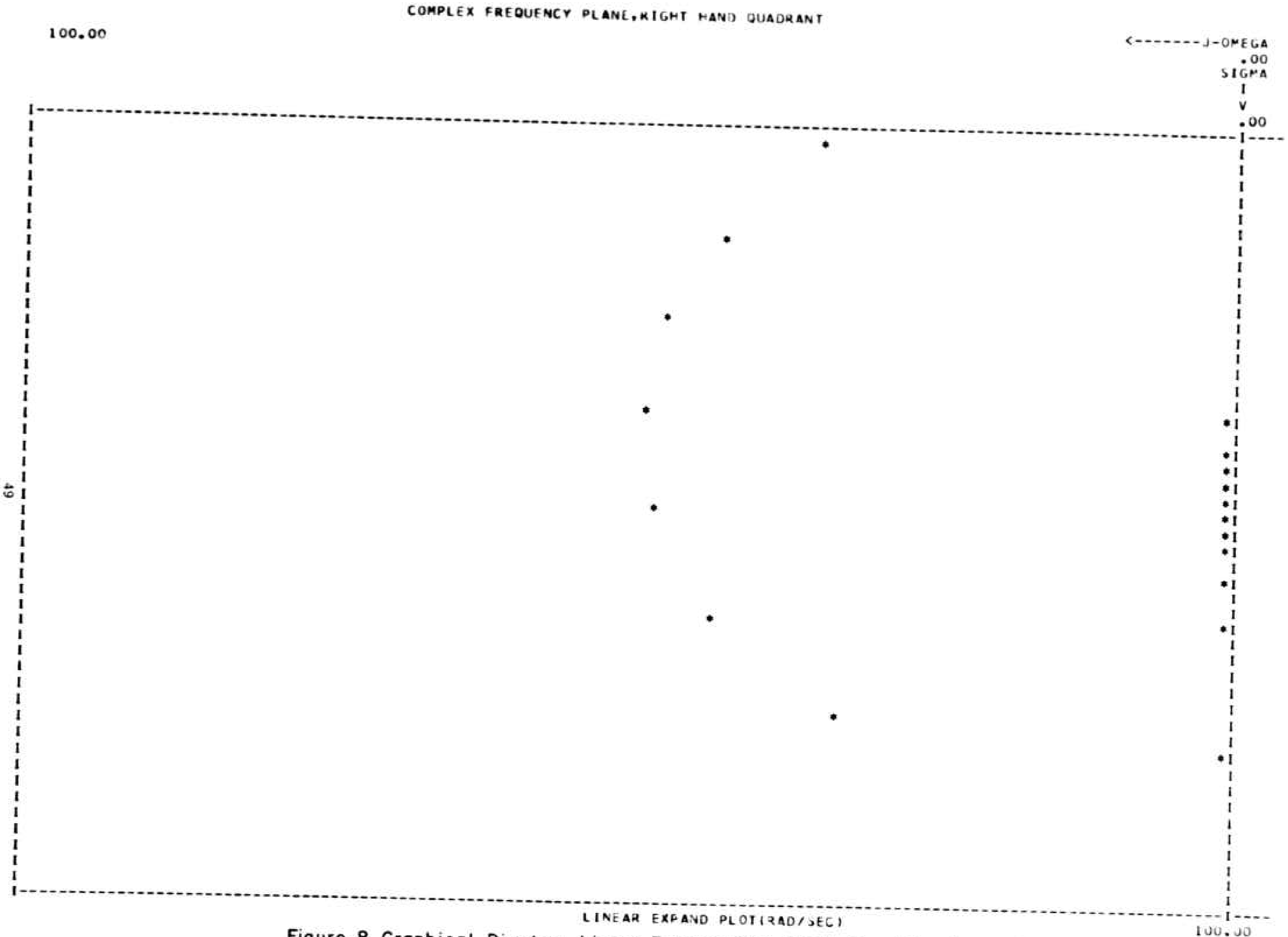

Figure 8. Graphical Display, Linear Expand Plot, Right Hand Quadrant, Example 1.

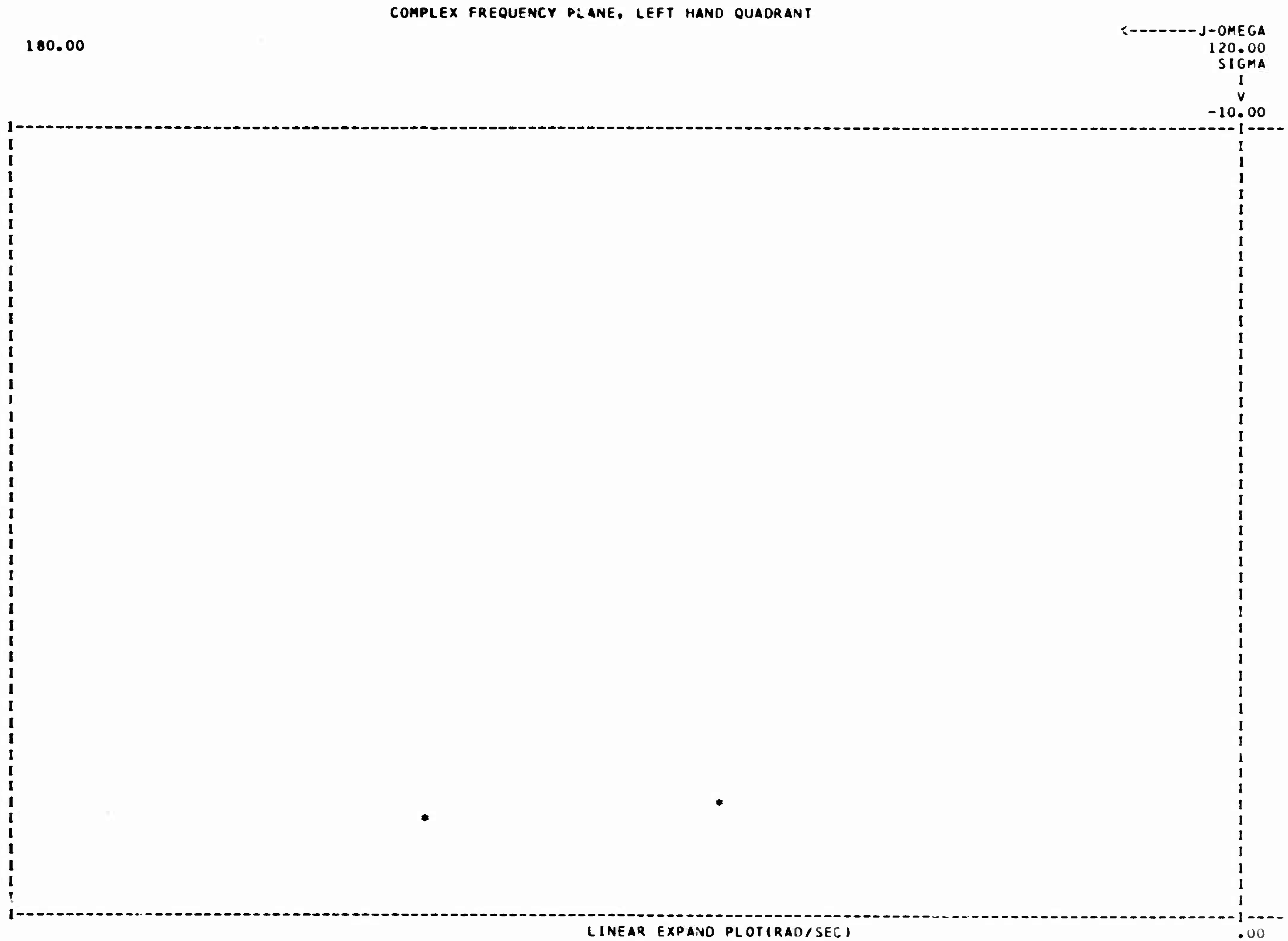

Figure 9. Graphical Display, Linear Expand Plot, Left Hand Quadrant, Example 1,

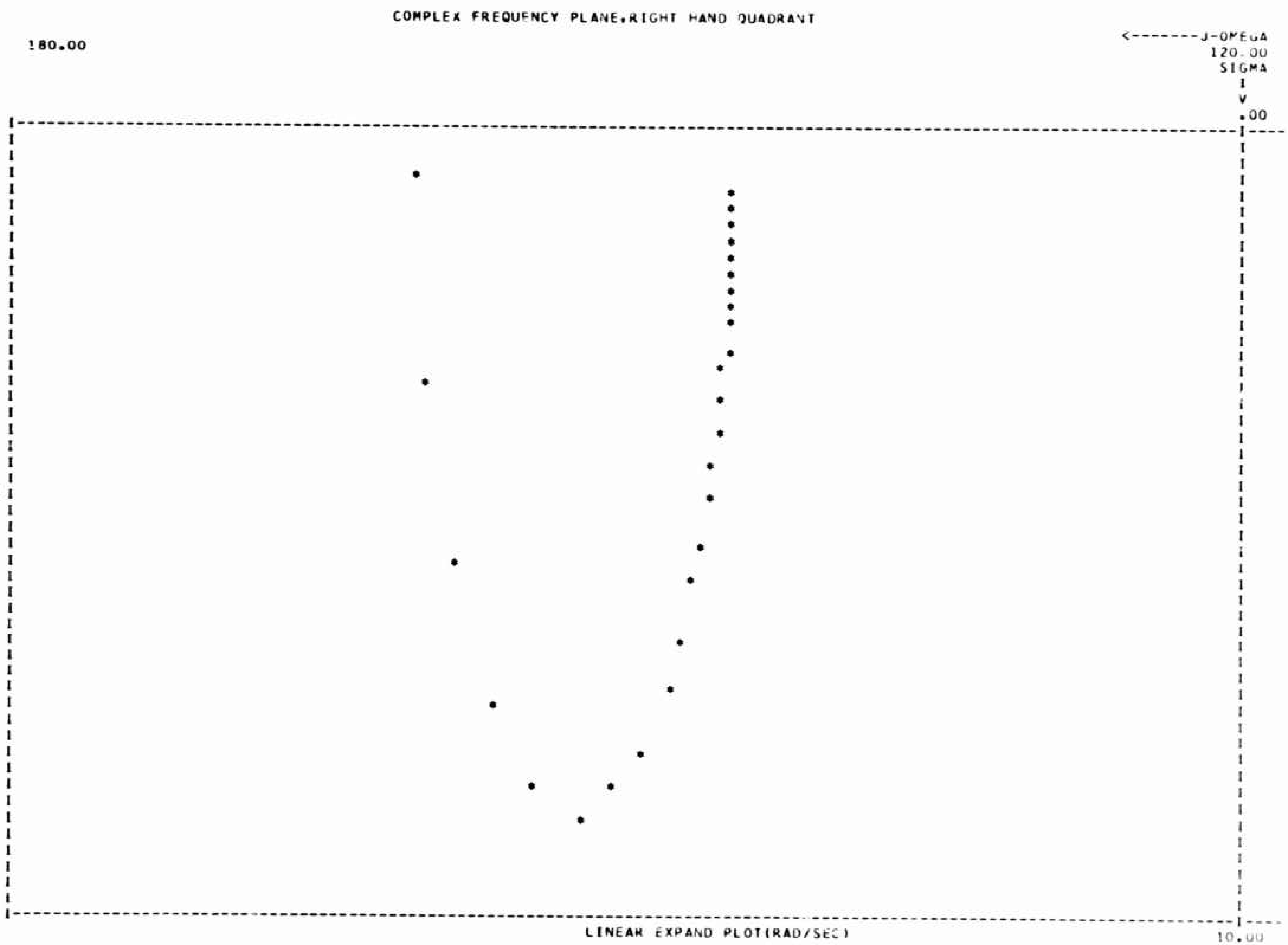

Figure 10. Graphical Display, Linear Expand Plot, Right Hand Quadrant, Example I.

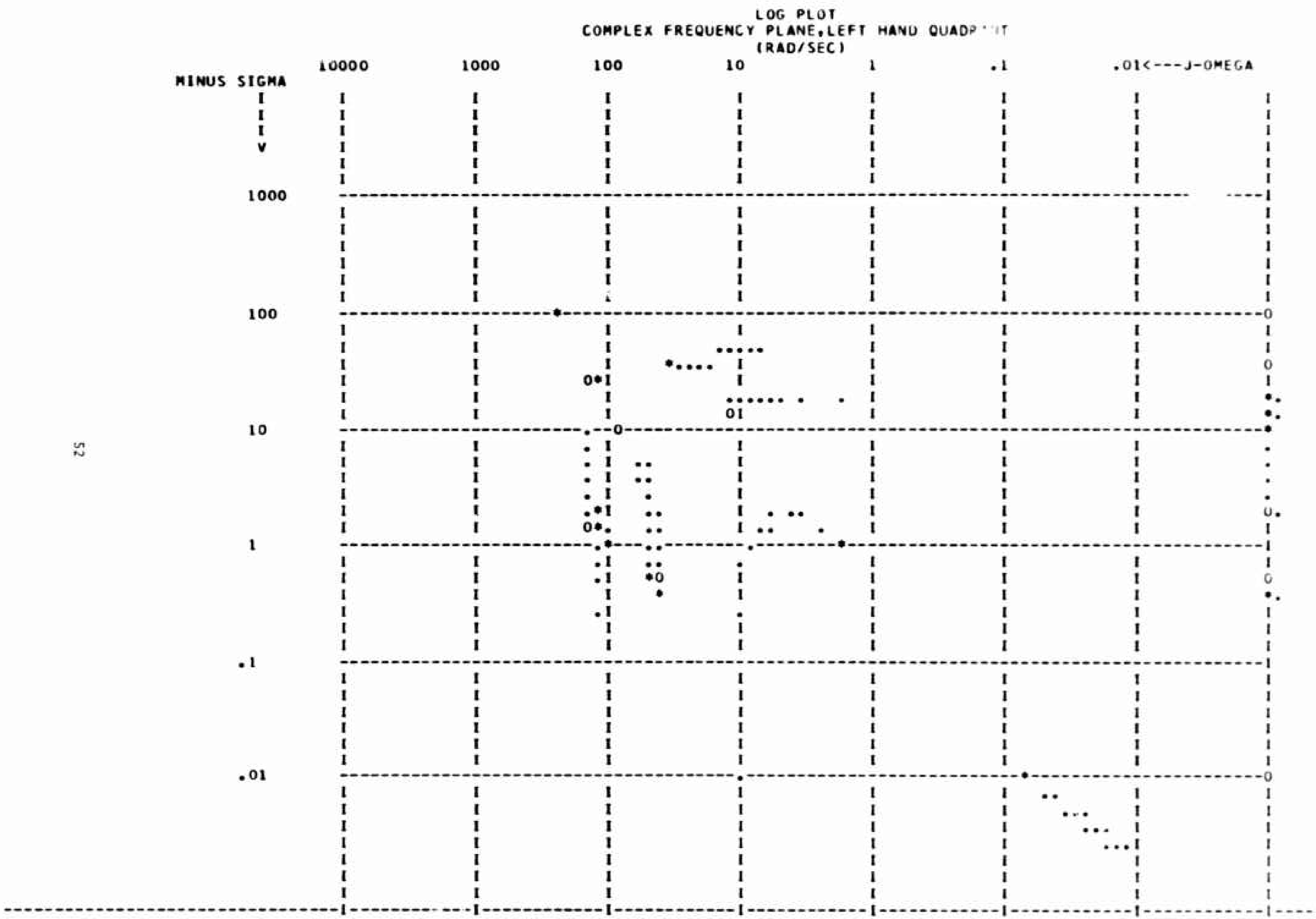

Figure 11. Graphical Display, Log Plot, Left Hand Quadrant, Example 2

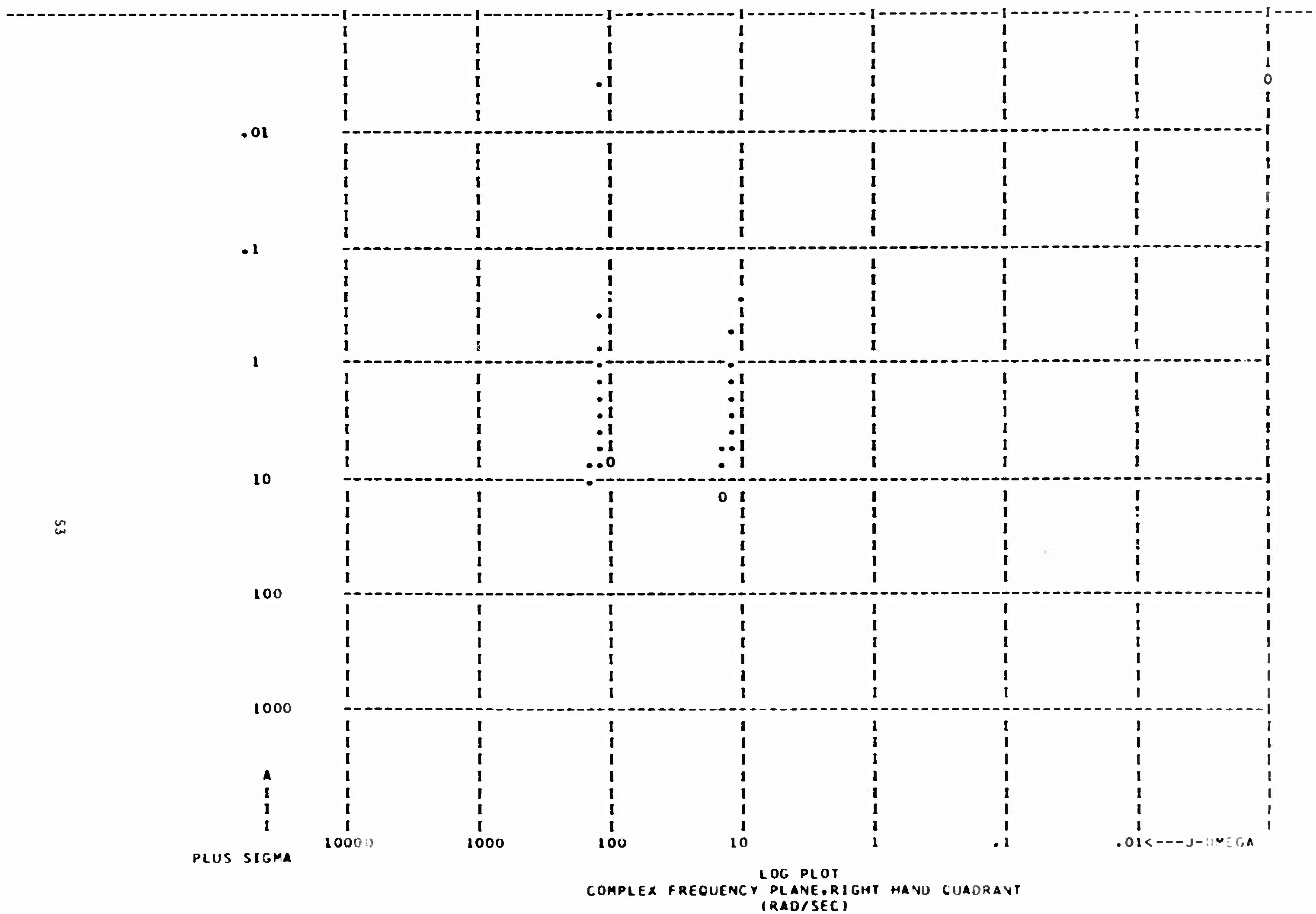

Figure 12. Graphical Display, Log Plot, Right Hand Quadrant, Example 2

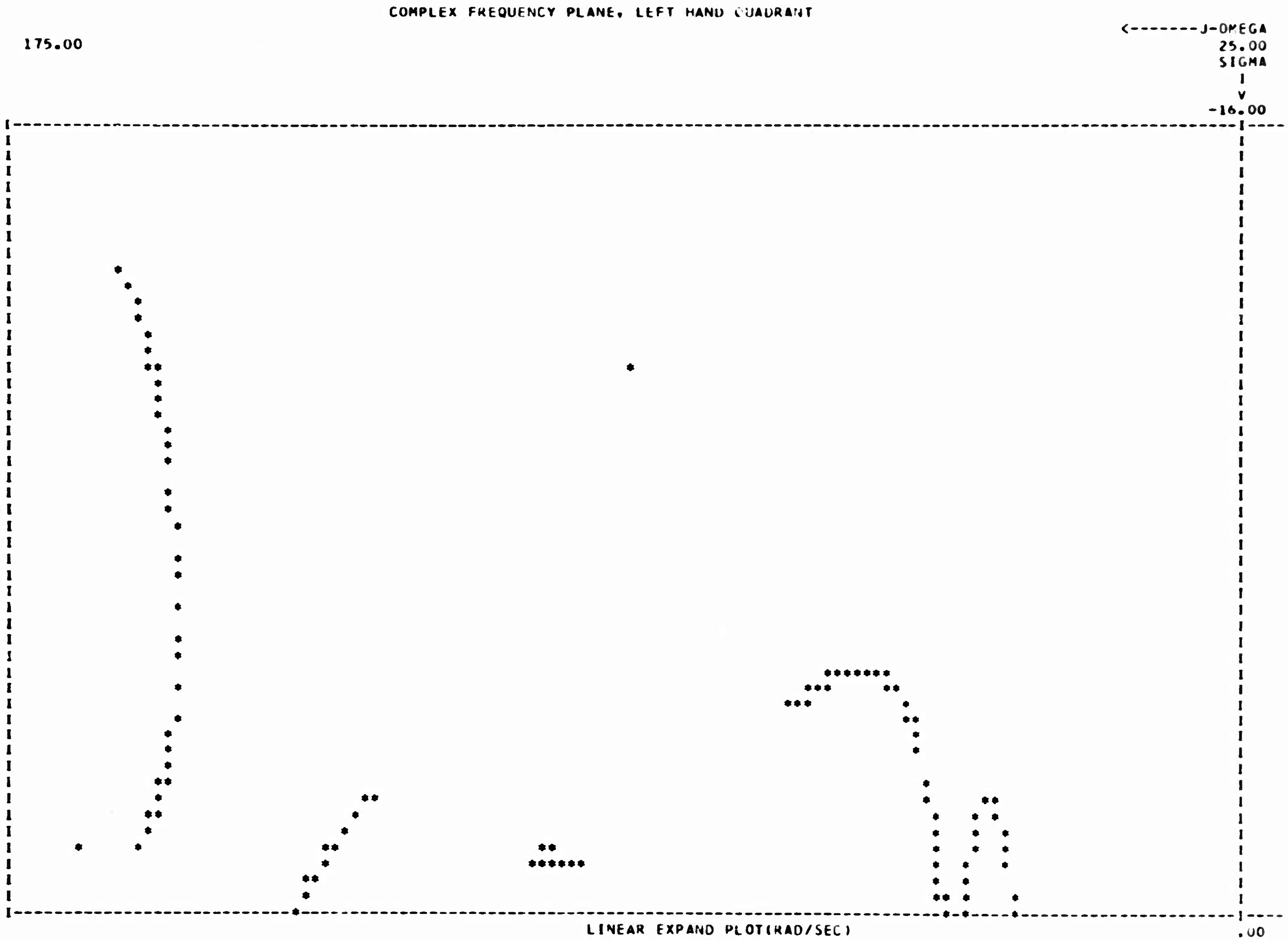

Figure 13. Graphical Display, Linear Expand Plot, Left Hand Quadrand, Example 2.

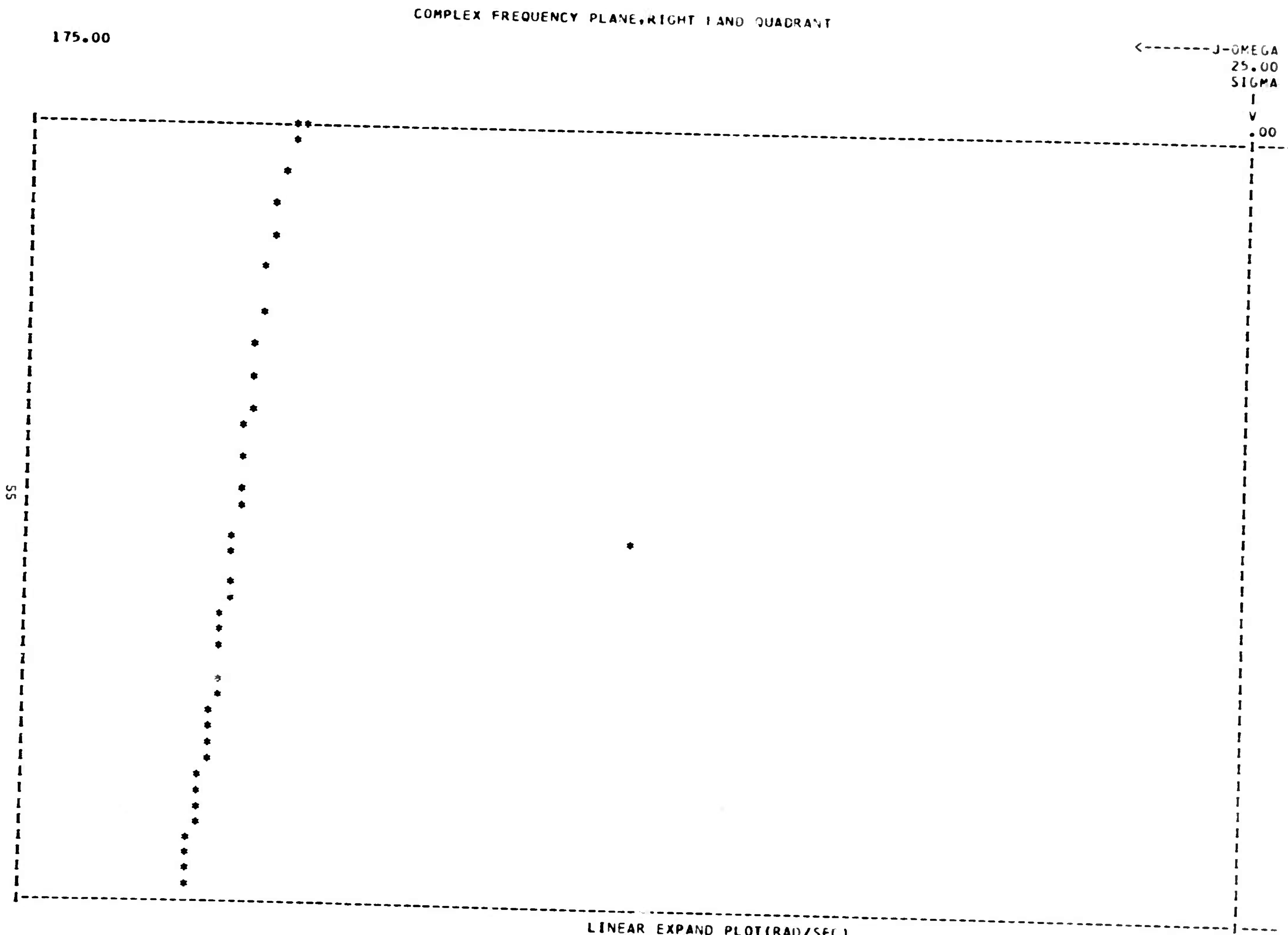

Figure 14 Graphical Display, Linear Expand Plot, Right Hand Quadrant, Example 2

VI. CONCLUSIONS

The method described in this memorandum for computing linear system root loci and plotting their trajectories on the complex frequency plane has the following merits.

1. Accepts input in the form of sums of products of polynomials.

2. Produces a log plot of the entire complex plane in two graphical displays.

3. Produces selectable scaled linear plots of regions in the complex frequency plane.

4. Relates the closed loop gain to the graphical display.

REFERENCES

1. Truxal, John G. "Control System Synthesis", McGraw Hill, N. Y. 1955, pp 221-277.
2. Digital Computer Program "Polynomial Multiplication and Root Locus", Martin Marietta, Baltimore, Maryland.
3. Blakelock, John H. "Automatic Control of Aircraft and Missiles", John Wiley & Sons, Inc., New York, 1965, pp. 306-327.

APPENDIX

LISTING OF SOURCE DECK

```
      CALL RTLOCS
      END
C
C
C
C
      SUBROUTINE RTLOCS
      DIMENSION WLOG(3000),PHILIN(3000),MDBLIN(3000),PHIPLN(3000),
     *MPDBLN(3000)
      DIMENSION SAVE1(100,100),SAVE2(100,100),XA(2000),XB(2000)
      DIMENSION X(100),IGA(100),IGB(100),C(100),D(100),ANS(100),
     1SAVE(100),ERA(100),AS(100),PS(100),A(100),B(100),ROOTR(100),
     2ROOTI(100),ATK(100),CK(100),CKS(100)
      INTEGER COMENT(20)
      INTEGER STAR1,STAR2,STAR3,STAR
      INTEGER ONE,ZERO,DOT,VEE,AYE
      INTEGER POINT(130,100),DASH,BLANK,STAR,Q,SPOT(130,50)
      REAL MDBLIN,MPDBLN
      DATA ONE/1H1/,ZERO/1HO/,DOT/1H./,AYE/1HA/,VEE/1HV/
      DATA BLANK/1H /,DASH/1H-/,          Q/1HI/
      DATA STAR1/1HO/,STAR2/1H*/,STAR3/1H./
      LOGICAL PRNT
      EQUIVALENCE (  SAVE1(1,1),WLOG(1)),(SAVE2(1,1),PHILIN(1)),
     1            (  SAVE1(1,31),MDBLIN(1)),(SAVE2(1,31),PHIPLN(1)),
     2            (  SAVE1(1,61),MPDBLN(1))
      EQUIVALENCE (SPOT(1,1),POINT(1,1))
      EQUIVALENCE(STAR1,ZERO),(STAR3,DOT)
      EQUIVALENCE(JACKIE,KJ,I),(JCANN,KK,J)
      COMMON/FREEK/A,B
      COMMON /BOB/ POINT
      COMMON /INFO4/ Q,BLANK,DASH
      COMMON /INFO5/ SAVE1,SAVE2
      READ(5,11) COMENT
      WRITE(6,13) COMENT
   13 FORMAT(///////////,20X,20A4)
   11 FORMAT(20A4)
      DO277 IOU=1,2000
      XA(IOU)=0.0
  277 XB(IOU)=0.0
      DO 401 KK=1,100
      DO 401 KJ=1,130
  401 POINT(KJ,KK)=BLANK
      DO 402 KK=1,130
      POINT(KK,51)=DASH
  402 POINT(KK,50)=DASH
      DO 403 KK=1,100
      DO 403 KJ=34,125,13
  403 POINT(KJ,KK)=Q
      DO 404 KJ=34,124
      DO 404 KK=7, 93,7
      IF(KK.EQ.49) KK=58
  404 POINT(KJ,KK)=DASH
      DO 405 KK=7, 93,7
      IF(KK.EQ.49) KK=58
  405 POINT(25,KK)=ONE
      DO 406 KK=26,28
  406 POINT(KK,7)=ZERO
      DO 407 KK=26,27
  407 POINT(KK,14)=ZERO
      POINT(26,21)=ZERO
```

```
      DO 408 KK=35,65,7
      IF(KK.EQ.49) KK=58
  408 POINT(24,KK)=DOT
      POINT(25,42)=ZERO
      POINT(25,58)=ZERO
      POINT(26,42)=ONE
      POINT(26,58)=ONE
      POINT(26,72)=ZERO
      POINT(26,79)=ZERO
      POINT(26,86)=ZERO
      POINT(27,86)=ZERO
      DO 409 KK=26,28
  409 POINT(KK,93)=ZERO
      DO 420 KK=1,100
      IF(KK.EQ.4) KK=98
  420 POINT(26,KK)=Q
      POINT(26,4)=VEE
      POINT(26,97)=AYE
      DO 7681 JACKIE=1,100
      DO 7681 JOANN=1,100
      SAVE1(JACKIE,JOANN)=0.0
 7681 SAVE2(JACKIE,JOANN)=0.0
      NO=1
   12 DO 5 I = 1,100
      A(I) = 0.0
5     B(I) = 0.0
      MI=0
      JZO=1
      K1=1
      III=1
      READ (5,101) N,IA,IB,IPROB
  140 FORMAT(10X,7HDELTAK=,I10,5X,19HPOLY.ADDED IN A(S)=,I10,5X,19HPOLY.
     *ADDED IN B(S)=,I10,5X,9HPROB.NO.=,I10)
      IF(IB.NE.0) GO TO 10
      IF(IA.NE.0) GO TO 10
      IF( N.NE.0) GO TO 10
      IF(IPROB.EQ.10000) GO TO 1016
 101  FORMAT (7I10)
   10 WRITE(6,1)
      FORK1=0.0
      FORK2 = 0.0
      FORK3 = 0.0
      WRITE (6 ,102)IPROB
  102 FORMAT (1H1, 9X,40HPOLYNOMIAL MULTIPLICATION AND ROOT LOCUS,44X,11
     1HPROBLEM NO.,I5//)
      IF (N) 21, 20, 21
   20 READ(5,103)Y,DY,YT
      WRITE(6,140)N,IA,IB,IPROB
      WRITE(6,141)Y,DY,YT
  141 FORMAT(1H0,5X,10HK-INITIAL=F20.10,5X,12HINCREMENT K=,F20.10,5X,12H
     *K-TERMINATE=,F20.10)
  103 FORMAT (4F10.0)
      GO TO 25
   21 READ(5,104)(X(I),I=1,N)
      WRITE(6,142) (I,X(I),I=1,N)
  104 FORMAT(7E10.0)
   25 READ (5 ,105)(IGA (I), I = 1, IA)
      WRITE(6,143) (I,IGA(I),I=1,IA)
  143 FORMAT(10X,24HNUMBER OF POLY. IN GROUP, I3,14H OF NUMERATOR=,I10)
  142 FORMAT(10X,2HX(,I3,2H)=,F20.10)
```

```
      READ (5 ,105)(IGB(J), J= 1, IB)
      WRITE(6,144) (J,IGB(J),J=1,IB)
  144 FORMAT(10X,24HNUMBER OF POLY. IN GROUP, I3,16H OF DENOMINATOR=,I10
     *)
105   FORMAT(7I10)
 300  DO 15 I =  1,100
15    SAVE(I) = 0.0
      ISDEG = 0
      JJ = 1
200   READ (5 ,106)MDEG,( C(I), I=1, 6)
      IF(MDEG.GT.6) GO TO 48
      WRITE(6,145) (I,C(I),I=1,MDEG)
      IF (MDEG - 6) 49,  49,  48
48    READ (5 ,104)(C(I), I = 7, MDEG)
      WRITE(6,145) (I,C(I),I=1,MDEG)
 106  FORMAT(I10,6E10.0)
49    IF (IGA(JJ)-1)  50, 51, 50
51    DO 56  I= 1, MDEG
56    ANS(I) = C(I)
      IADEG = MDEG
      IF (ISDEG - MDEG)  52, 52, 53
53    INDEG = ISDEG
      GO TO 68
52    INDEG = MDEG
      GO TO 68
50    READ (5 ,106)NDEG, (D(I), I =1, 6)
      IF(NDEG.GT.6) GO TO 54
      WRITE(6,145) (I,D(I),I=1,NDEG)
      IF (NDEG - 6)  55, 55, 54
54    READ (5 ,104)(D(I), I=7, NDEG)
      WRITE(6,145) (I,D(I),I=1,NDEG)
55    IADEG = NDEG + MDEG -1
      CALL POLMPY (C,MDEG,D,NDEG,ANS)
      IGA (JJ) = IGA(JJ) -1
      IF (IGA(JJ) -1) 65, 65, 64
64    DO 60  I = 1, IADEG
60    C(I) = ANS(I)
      MDEG =IADEG
      GO TO 50
65    IF (ISDEG - IADEG) 66,66,67
66    INDEG = IADEG
      GO TO 68
67    INDEG = ISDEG
68    CALL POLADD (SAVE,ISDEG,ANS,IADEG,ERA)
  145 FORMAT( 10X,2HC(,I3,2H)=,F20.10)
      WRITE(6,1)
    1 FORMAT(1H1//)
6800  IF (ERA(INDEG))6803, 6802, 6803
6802  INDEG = INDEG - 1
      IF (INDEG)  6801, 6801, 6800
6801  INDEG = 1
6803  JJ = JJ + 1
      DO 70 I = 1, INDEG
70    SAVE(I) = ERA(I)
      ISDEG = INDEG
      IA = IA -1
      IF (IA) 201, 201, 200
 201  IF (FORK1) 202, 202, 203
C     SAVE NUMERATOR.
 202  DO 220 I = 1, ISDEG
```

```
 220  A(I) = SAVE(I)
      IDA = ISDEG
      IA = IB
      FORK1 = 1.0
      DO 230 I=1, IA
 230  IGA(I) = IGB(I)
C     START DENOMINATOR
      GO TO 300
C     SAVE DENOM.
 203  DO 240 I = 1, ISDEG
 240  B(I) = SAVE(I)
      IDB = ISDEG
      WRITE (6 ,109)
 109  FORMAT (10X,41HCOEFFICIENTS ARE GIVEN IN ASCENDING ORDER/////)
 339  IF (A(IDA)) 340, 341, 340
 341  IDA = IDA - 1
      IF (IDA)  345, 345, 339
 345  WRITE (6 ,120)
 120  FORMAT (1H0,10X,20HPOLYNOMIAL A IS ZERO//)
      FORK2 = 1.0
      GO TO 410
 340  IF (IDA - 2) 346, 347, 335
  346 WRITE(6,121)A(1)
      STAR=STAR1
      PRNT=.TRUE.
      ANUMB1=A(1)
      ANUMB2=0.0
      CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III
     *,NO)
 121  FORMAT (1H0,10X,28HPOLYNOMIAL A IS A CONSTANT =,1P1E16.7//)
      GO TO  410
 347  ROOT = - A(1) / A(2)
      WRITE (6 ,133)A(1), A(2)
 133  FORMAT (10X,21HTHE COEFFICIENTS OF A/1P2E20.7)
      WRITE (6 ,122)ROOT
      STAR=STAR1
      PRNT=.TRUE.
      ANUMB1=ROOT
      ANUMB2=0.0
      CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III
     *,NO)
 122  FORMAT (1H0,10X,23HROOT OF POLYNOMIAL A IS,1P1E16.7//)
      GO TO  410
C     WRITE POLYS
 335  ID1A =IDA -1
      WRITE (6,107)ID1A,(A(I),I=1,IDA)
      K = IDA
      DO 800 I = 1,IDA
      AS(I) = A(K)
 800  K = K-1
      IDP2A=IDA *2
      ID2A= 2 *ID1A
      CALL MULLER (AS,ID1A,ROOTR,ROOTI)
      DO 805 I = 1,ID1A
      SAM = 100. * AMAX1(ABS(ROOTR(I)),ABS(ROOTI(I)))
      IF (SAM + ABS(ROOTR(I)).EQ. SAM) ROOTR(I)= 0.0
      IF (SAM + ABS(ROOTI(I)).EQ. SAM) ROOTI(I)= 0.0
  805 CONTINUE
 400  WRITE (6,111) (ROOTR(I),ROOTI(I),I=1,ID1A)
      CALL ERCHEK(ROOTI,ID1A)
```

```
      PRNT=.TRUE.
      STAR=STAR1
      DO 2 III=1,ID1A
      ANUMB1=ROOTR(III)
      ANUMB2=ABS(ROOTI(III))
      CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III
     *,NO)
    2 CONTINUE
410   IF(B(IDB)) 411, 412, 411
412   IDB = IDB - 1
      IF (IDB)  445, 445, 410
445   WRITE (6 ,123)
123   FORMAT (1H0,10X,20HPOLYNOMIAL B IS ZERO//)
      IF (FORK2)12,450,12
450   FORK3 = 1.0
      GO TO 698
411   IF (IDB - 2)  451, 452, 499
451   WRITE (6 ,124)B(IDB)
      STAR=STAR2
      PRNT=.TRUE.
      ANUMB1=B(IDB)
      ANUMB2=0.0
      CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III
     *,NO)
      PRNT=.FALSE.
      STAR=STAR3
124   FORMAT (1H0,10X,28HPOLYNOMIAL B IS A CONSTANT =,1P1E16.7//)
      GO TO 698
452   ROOT = -B(1) / B(2)
      WRITE (6 ,134)B(1), B(2)
134   FORMAT (10X,21HTHE COEFFICIENTS OF B/1P2E20.7)
      WRITE (6 ,125)ROOT
      STAR=STAR2
      PRNT=.TRUE.
      ANUMB1=ROOT
      ANUMB2=0.0
      CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III
     *,NO)
      PRNT=.FALSE.
      STAR=STAR3
125   FORMAT (1H0,10X,23HROOT OF POLYNOMIAL B IS,1P1E16.7//)
      GO TO 698
  107 FORMAT (10X,42HTHE COEFFICIENTS OF POLYNOMIAL A (ORDER = I3,1H)/ (
     11P6E20.7))
  499 ID1B = IDB -1
      WRITE (6,108)ID1B,(B(I),I=1,IDB)
  108 FORMAT (////10X,42HTHE COEFFICIENTS OF POLYNOMIAL B (ORDER = I3,1H
     1)/ (1P6E20.7))
      K = IDB
      DO 801 I = 1,IDB
      BS(I) = B(K)
801   K = K-1
      IDP2B= IDB * 2
      ID2B = 2 * ID1B
      CALL MULLER (BS,ID1B,ROOTR,ROOTI)
      DO 806 I = 1,ID1B
      SAM = 100. * AMAX1(ABS(ROOTR(I)),ABS(ROOTI(I)))
      IF (SAM + ABS(ROOTR(I)).EQ. SAM) ROOTR(I)= 0.0
      IF (SAM + ABS(ROOTI(I)).EQ. SAM) ROOTI(I)= 0.0
  806 CONTINUE
```

```
 500  WRITE (6,112)(ROOTR(I),ROOTI(I),I= 1,ID1B)
      CALL ERCHEK(ROOTI,ID1B)
      STAR=STAR2
      DO 3 III=1,ID1B
      ANUMB1=ROOTR(III)
      ANUMB2=ABS(ROOTI(III))
      CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III
     *,NO)
    3 CONTINUE
      PRNT=.FALSE.
      STAR=STAR3
  111 FORMAT (1H0,11X,14HTHE ROOTS OF A/ (1P1E20.7,6H    +I ,1P1E14.7,1P1
     1E20.7,6H    +I ,1P1E14.7,1P1E20.7,6H    +I ,1P1E14.7))
  112 FORMAT (1H0,11X,14HTHE ROOTS OF B/ (1P1E20.7,6H    +I ,1P1E14.7,1P1
     1E20.7,6H    +I ,1P1E14.7,1P1E20.7,6H    +I ,1P1E14.7))
  698 IF (FORK2)12,699,12
  699 IF (FORK3)12,6991,12
 6991 WRITE (6 ,102)IPROB
      MSHEET = 5
C     START K CALCULATIONS
      IF (N) 702,702,533
  533 DO 550 I= 1, N
      DO 541 J= 1, IDA
  541 ATK(J) =  X(I) * A(J)
C     COMPUTE ROOTS OF K * A + B
      IDC= MAX0( IDA, IDB)
      CALL POLADD (ATK,IDA,B,IDB,CK)
      IDS = IDC
  554 IF (CK(IDS))555, 557, 555
  557 IDS = IDS - 1
      IF (IDS) 558,558, 554
  558 WRITE (6 ,129)X(I)
  129 FORMAT (1H0,10X,35HPOLYNOMIAL K*A + B  IS ZERO FOR K =,1P1E16.7//)
      GO TO 550
  555 IF (IDS - 2)  559, 560, 561
  559 WRITE (6 ,130)CK(IDS), X(I)
  130 FORMAT (1H0,10X,35HPOLYNOMIAL K*A + B IS A CONSTANT = ,1P1E15.7,10
     1H FOR K =  ,1P1E14.7//)
      GO TO 550
  560 ROOT = -CK(1) / CK(2)
      WRITE (6 ,131)ROOT, X(I)
  131 FORMAT (1H0,10X,18HROOT OF K*A + B = ,1P1E15.7,10H FOR K =  ,1P1E1
     14.7//)
      GO TO 550
  561 K = IDS
      DO 803 J = 1,IDS
      CKS(J) = CK(K)
  803 K = K - 1
      ID1C = IDS - 1
      IDP2C = IDS * 2
      ID2C  = 2  * ID1C
      CALL MULLER (CKS,ID1C,ROOTR,ROOTI)
      DO 807 J = 1,ID1C
      SAM = 100. * AMAX1(ABS(ROOTR(J)),ABS(ROOTI(J)))
      IF (SAM + ABS(ROOTR(J)).EQ.SAM)  ROOTR(J) = 0.0
      IF (SAM + ABS(ROOTI(J)).EQ.SAM)  ROOTI(J) = 0.0
  807 CONTINUE
      WRITE (6,808)ID1C,X(I),(CK(J),J=1,IDS)
  808 FORMAT (///10X,48HTHE COEFFICIENTS OF POLYNOMIAL K*A + B (ORDER =
     1I3,7H)  K = 1P1E16.7/(1P6E20.7))
```

```
  545 WRITE (6,115)(ROOTR(J),ROOTI(J),J=1,ID1C)
      CALL ERCHEK(ROOTI,ID1C)
      CALL SAVER(ROOTR,ROOTI,ID1C,SAVE1,SAVE2,JZO,K1)
  115 FORMAT (1HO,9X,16HROOTS OF K*A + B/(1P1E20.7,6H   + I ,1P1E14.7,1P1
     1E20.7,6H   + I ,1P1E14.7,1P1E20.7,6H   + I ,1P1E14.7))
 5452 MSHEET = MSHEET - 1
      IF (MSHEET) 546, 546, 550
  546 WRITE (6 ,102)IPROB
      MSHEET = 5
  550 CONTINUE
      GO TO 12
  702 DO  705   J = 1, IDA
  705 ATK(J) = Y * A(J)
C     COMPUTE ROOTS OF K * A + B
      IDC= MAXO(IDA, IDB)
      CALL POLADD (ATK,IDA,B,IDB,CK)
      IDS = IDC
  754 IF (CK(IDS))755, 757, 755
  757 IDS = IDS - 1
      IF (IDS) 758, 758, 754
  758 WRITE (6 ,129)Y
      GO TO 711
  755 IF (IDS - 2) 759, 760, 761
  759 WRITE (6 ,130)CK(IDS), Y
      GO TO 711
  760 ROOT = -CK(1) / CK(2)
      WRITE (6 ,131)ROOT, Y
      GO TO 711
  761 K = IDS
      DO 804 I = 1,IDS
      CKS(I) = CK(K)
  804 K = K -1
      ID1C = IDS - 1
      IDP2C = IDS * 2
      ID2C  = 2 * ID1C
      CALL MULLER (CKS,ID1C,ROOTR,ROOTI)
      DO 809 I = 1,ID1C
      SAM = 100. * AMAX1(ABS(ROOTR(I)),ABS(ROOTI(I)))
      IF (SAM + ABS(ROOTR(I)).EQ. SAM) ROOTR(I)= 0.0
      IF (SAM + ABS(ROOTI(I)).EQ. SAM) ROOTI(I)= 0.0
  809 CONTINUE
      WRITE (6,808)ID1C,Y,(CK(I),I=1,IDS)
      WRITE (6,115) (ROOTR(J),ROOTI(J),J=1,ID1C)
      CALL ERCHEK(ROOTI,ID1C)
      CALL SAVER(ROOTR,ROOTI,ID1C,SAVE1,SAVE2,JZO,K1)
  711 Y = Y + DY
      IF (Y - YT) 712,712,12
  712 MSHEET = MSHEET - 1
      IF (MSHEET) 713, 713, 702
  713 WRITE (6 ,102)IPROB
      MSHEET = 5
      GO TO 702
 1016 CALL PLOTER(SAVE1,SAVE2,ANUMB1,ANUMB2,POINT,XA,XB,MI,PRNT,STAR,III
     *,NO)
      READ(5,22)L
   22 FORMAT(I10)
      IF(L.EQ.0) GO TO 1017
      CALL EXPAND(L,XA,XB)
 1017 RETURN
      END
```

```
C
C
C
C
      SUBROUTINE POLMPY (A,N,B,M,C)
      DIMENSION A(1),B(1),C(1)
      K = M+N
      DO 5 I=1,K
    5 C(I) = 0.0
      DO 10 I=1,N
      L = I-1
      DO 10 J=1,M
      L = L+1
   10 C(L) = C(L)+A(I)*B(J)
      RETURN
      END
C
C
C
C
      SUBROUTINE POLADD (A,N,B,M,C)
      DIMENSION A(1),B(1),C(1)
      IF (N-M) 1,1,2
    1 NK = N
      GO TO 5
    2 NK = M
    5 DO 10 I=1,NK
   10 C(I) = A(I)+B(I)
      NK = NK+1
      IF (N-M) 11,25,15
   11 DO 20 I=NK,M
   20 C(I) = B(I)
   25 RETURN
   15 DO 30 I=NK,N
   30 C(I) = A(I)
      RETURN
      END
C
C
C
C
      SUBROUTINE MULLER(COE,N1,ROOTR,ROOTI)
       DIMENSION COE(1),ROOTR(1),ROOTI(1)
      N2=N1+1
      N4=0
      I=N1+1
   19 IF(COE(I))9,7,9
    7 N4=N4+1
      ROOTR(N4)=0.
      ROOTI(N4)=0.
      I=I-1
      IF(N4-N1)19,37,19
    9 CONTINUE
   10 AXR=0.8
      AXI=0.
      L=1
      N3=1
      ALP1R=AXR
      ALP1I=AXI
      M=1
```

```
      GOTO99
   11 BET1R=TEMR
      BET1I=TEMI
      AXR=0.85
      ALP2R=AXR
      ALP2I=AXI
      M=2
      GOTO99
   12 BET2R=TEMR
      BET2I=TEMI
      AXR=0.9
      ALP3R=AXR
      ALP3I=AXI
      M=3
      GOTO99
   13 BET3R=TEMR
      BET3I=TEMI
   14 TE1=ALP1R-ALP3R
      TE2=ALP1I-ALP3I
      TE5=ALP3R-ALP2R
      TE6=ALP3I-ALP2I
      TEM=TE5*TE5+TE6*TE6
      TE3=(TE1*TE5+TE2*TE6)/TEM
      TE4=(TE2*TE5-TE1*TE6)/TEM
      TE7=TE3+1.
      TE9=TE3*TE3-TE4*TE4
      TE10=2.*TE3*TE4
      DE15=TE7*BET3R-TE4*BET3I
      DE16=TE7*BET3I+TE4*BET3R
      TE11=TE3*BET2R-TE4*BET2I+BET1R-DE15
      TE12=TE3*BET2I+TE4*BET2R+BET1I-DE16
      TE7=TE9-1.
      TE1=TE9*BET2R-TE10*BET2I
      TE2=TE9*BET2I+TE10*BET2R
      TE13=TE1-BET1R-TE7*BET3R+TE10*BET3I
      TE14=TE2-BET1I-TE7*BET3I-TE10*BET3R
      TE15=DE15*TE3-DE16*TE4
      TE16=DE15*TE4+DE16*TE3
      TE1=TE13*TE13-TE14*TE14-4.*(TE11*TE15-TE12*TE16)
      TE2=2.*TE13*TE14-4.*(TE12*TE15+TE11*TE16)
       TEM = SQRT (TE1*TE1+TE2*TE2)
      IF(TE1)113,113,112
  113  TE4 = SQRT (.5 * (TEM - TE1))
      TE3=.5*TE2/TE4
      GO TO 111
  112  TE3 = SQRT (.5 * (TEM + TE1))
      IF(TE2)110,200,200
  110 TE3=-TE3
  200 TE4=.5*TE2/TE3
  111 TE7=TE13+TE3
      TE8=TE14+TE4
      TE9=TE13-TE3
      TE10=TE14-TE4
      TE1=2.*TE15
      TE2=2.*TE16
      IF(TE7*TE7+TE8*TE8-TE9*TE9-TE10*TE10)204,204,205
  204 TE7=TE9
      TE8=TE10
  205 TEM=TE7*TE7+TE8*TE8
      TE3=(TE1*TE7+TE2*TE8)/TEM
```

```
      TE4=(TE2*TE7-TE1*TE8)/TEM
      AXR=ALP3R+TE3*TE5-TE4*TE6
      AXI=ALP3I+TE3*TE6+TE4*TE5
      ALP4R=AXR
      ALP4I=AXI
      M=4
      GO TO 99
   15 N6=1
   38  IF (ABS (HELL) + ABS (BELL) - 1.E-20) 18,18,16
   16  TE7 = ABS (ALP3R - AXR) + ABS (ALP3I - AXI)
       IF (TE7 / (ABS (AXR) + ABS (AXI)) - 1.E-7)18,18,17
   17 N3=N3+1
      ALP1R=ALP2R
      ALP1I=ALP2I
      ALP2R=ALP3R
      ALP2I=ALP3I
      ALP3R=ALP4R
      ALP3I=ALP4I
      BET1R=BET2R
      BET1I=BET2I
      BET2R=BET3R
      BET2I=BET3I
      BET3R=TEMR
      BET3I=TEMI
      IF(N3-100)14,18,18
   18 N4=N4+1
      ROOTR(N4)=ALP4R
      ROOTI(N4)=ALP4I
      N3=0
   41 IF(N4-N1)30,37,37
   37 RETURN
   30  IF (ABS (ROOTI(N4)) - 1.E-5)10,10,31
   31 GO TO(32,10),L
   32 AXR=ALP1R
      AXI=-ALP1I
      ALP1I=-ALP1I
      M=5
      GO TO 99
   33 BET1R=TEMR
      BET1I=TEMI
      AXR=ALP2R
      AXI=-ALP2I
      ALP2I=-ALP2I
      M=6
      GO TO 99
   34 BET2R=TEMR
      BET2I=TEMI
      AXR=ALP3R
      AXI=-ALP3I
      ALP3I=-ALP3I
      L=2
      M=3
   99 TEMR=COE(1)
      TEMI=0.0
      DO100I=1,N1
      TE1=TEMR*AXR-TEMI*AXI
      TEMI=TEMI*AXR+TEMR*AXI
  100 TEMR=      TE1+COE(I+1)
      HELL=TEMR
      BELL=TEMI
```

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```
   42 IF(N4)102,103,102
  102 DO101I=1,N4
      TEM1=AXR-ROOTR(I)
      TEM2=AXI-ROOTI(I)
      TE1=TEM1*TEM1+TEM2*TEM2
      TE2=(TEMR*TEM1+TEMI*TEM2)/TE1
      TEMI=(TEMI*TEM1-TEMR*TEM2)/TE1
  101 TEMR=TE2
  103 GO TO(11,12,13,15,33,34),M
      END
C
C
C
C
      SUBROUTINE ERCHEK(X,I)
      DIMENSION X(100)
      DATA ERRLIM/0.1E-8/
      DO 10 J=1,I
   10 IF(ABS(X(J)).LT.ERRLIM) X(J)=0.0
      RETURN
      END
C
C
C
C
      SUBROUTINE EXPAND(ILOVE,X,Y)
      DIMENSION X2(2000),Y2(2000) ,X(2000),Y(2000)
      DIMENSION X1(2000),Y1(2000),Z(125),Z1(87)
      INTEGER SPOT(130,50),DASH,Q,BLANK,POINT(130,100)
      DATA Q/1HI/, DASH/1H-/,BLANK/1H /
      DATA APLUS/0.0/,BPLUS/0.0/,AMINUS/0.0/,BMINUS/0.0/
      COMMON /INFO4/ Q,BLANK,DASH
      COMMON /BOB/POINT
      COMMON /PAYNE/AM,AP,BM,BP
      COMMON /PJ/X1,Y1
      EQUIVALENCE(X1(1),Z(1)),(Y1(1),Z1(1))
      EQUIVALENCE (SPOT(1,1),POINT(1,1)),(I1,I2)
      EQUIVALENCE(AP,APLUS),(AM,AMINUS),(BM,BMINUS),(BP,BPLUS)
      DO 40 J=1,ILOVE
      DO 50 M=1,2000
      X1(M)=0.0
   50 Y1(M)=0.0
      DO 51 M=1,130
      DO 51 N=1,50
   51 SPOT(M,N)=BLANK
      DO 52 M=1,130
      SPOT(M,1)=DASH
   52 SPOT(M,50)=DASH
      DO 53 M=1,50
      SPOT(1,M)=Q
   53 SPOT(126,M)=Q
      DO 54 M=1,2000
      X2(M)=0.0
   54 Y2(M)=0.0
      READ(5,10) OMEGA,ENCRMT,SIGMA,DELTA
      OMEGA=ABS(OMEGA)
   10 FORMAT(4F10.0)
      A=ENCRMT
      PERCNT=A*0.01
      B=OMEGA
```

```
      B=B*PERCNT
      APLUS=B+OMEGA
      AMINUS=OMEGA-B
      C=APLUS
      D=AMINUS
      I1=1
      DO 41 L=1,2000
      IF(Y(L).LT.D)GO TO 41
      IF(Y(L).GT.C)GO TO 41
   23 X1(I1)=X(L)
      Y1(I1)=Y(L)
      I1=I1+1
   41 CONTINUE
      A=DELTA
  901 PERCNT=A*0.01
      B=SIGMA
  903 B=B*PERCNT
      BPLUS=SIGMA+B
      BMINUS=SIGMA-B
      C=BPLUS
      D=BMINUS
      I2=1
  910 DO 42 L=1,2000
      IF(C.GT.0.0) GO TO 60
      IF(X1(L).GT.D) GO TO 42
      IF(X1(L).LT.C) GO TO 42
      GO TO 25
   60 IF(X1(L).GT.C) GO TO 42
      IF(X1(L).LT.D) GO TO 42
   25 X2(I2)=X1(L)
      Y2(I2)=Y1(L)
      I2=I2+1
   42 CONTINUE
      CALL SPLIT(X2,Y2,SPOT,APLUS,AMINUS,BPLUS,BMINUS)
   40 CONTINUE
      RETURN
      END
C
C
C
C
      SUBROUTINE SPLIT(X,Y,SPOT,APLUS,AMINUS,BPLUS,BMINUS)
      DIMENSION X1(2000),Y1(2000)
      DIMENSION X(2000),Y(2000),Z(125),Z1(87)
      INTEGER SPOT(130,50)
      COMMON /PJ/X1,Y1
      EQUIVALENCE(X1(1),Z(1)),(Y1(1),Z1(1))
      A=APLUS
      B=AMINUS
      C=A-B
      D=BPLUS
      E=BMINUS
      G=ABS(D)
      H=ABS(E)
      F=G-H
      DELTA=C/124.0
      DIFF=F/50.0
      DO 11 J=1,124
      Z(J)=A
   11 A=A-DELTA
```

```
      Z(125)=B
      IF(D.LT.0.0)DIFF=-DIFF
      DO 12 J=1,49
      Z1(J)=D
   12 D=D-DIFF
      Z1(50)=E
      CALL BRAKUP(Z,X,Y,SPOT,Z1)
      RETURN
      END
C
C
C
C
      SUBROUTINE BRAKUP(YY,X,Y,SPOT,XX)
      LOGICAL SKIP,SKIP1
      INTEGER SPOT(130,50),                     XPT,YPT,STAR
      DATA STAR/1H*/
      DIMENSION X(2000),Y(2000),YY(125),XX(87)
      COMMON /PAYNE/AM,AP,BM,BP
      EQUIVALENCE(AP,APLUS),(AM,AMINUS),(BM,BMINUS),(BP,BPLUS)
      SKIP=.FALSE.
      SKIP1=.FALSE.
      DO 43 J=1,2000
      DO 43 I=1,125
      IF(Y(J).NE.0.0) GO TO 30
      IF(X(J).NE.0.0) GO TO 30
      L=J+1
      M=J+6
      DO 60 N=L,M
      IF(N.GT.2000) GO TO 60
      IF(Y(N).NE.0.0) GO TO 30
      IF(X(N).NE.0.0) GO TO 30
   60 CONTINUE
      GO TO 40
   30 IF(SKIP) GO TO 20
      IF(Y(J).LT.YY(I)) GO TO 20
      YPT=I
      SKIP=.TRUE.
   20 IF(SKIP1)GO TO 48
      IF(I.GT.50) GO TO 43
      IF(BPLUS.GT.0.0) GO TO 10
      IF(X(J).GT.XX(I)) GO TO 43
      GO TO 11
   10 IF(X(J).LT.XX( I)) GO TO 43
   11 IF(BPLUS.LT.0.0) XPT=I
      IF(BPLUS.GE.0.) XPT=51-I
      SKIP1=.TRUE.
      IF(.NOT.SKIP)GO TO 43
   50 I=125
      SPOT(YPT,XPT)=STAR
      SKIP=.FALSE.
      SKIP1=.FALSE.
   48 IF(SKIP)GO TO 50
   43 CONTINUE
   40 CALL RITEIT(SPOT)
      RETURN
      END
C
C
C
```

```
C
      SUBROUTINE RITEIT(SPOT)
      COMMON /PAYNE/AM,AP,BM,BP
      EQUIVALENCE(AP,APLUS),(AM,AMINUS),(BM,BMINUS),(BP,BPLUS)
      INTEGER SPOT(130,50)
      IF(BP.LT.BM) GO TO 40
      A=BP
      B=BM
      WRITE(6,1)
    1 FORMAT(1H1,40X,43HCOMPLEX FREQUENCY PLANE,RIGHT HAND QUADRANT    )
      GO TO 30
   40 A=BM
      B=BP
      WRITE(6,2)
    2 FORMAT(1H1,40X,43HCOMPLEX FREQUENCY PLANE, LEFT HAND QUADRANT    )
   30 WRITE(6,12)AP,AM
   12 FORMAT(114X,15H<-------J-OMEGA,/1X,F8.2,112X,F8.2)
      WRITE(6,14)B
   14 FORMAT(124X,5HSIGMA,/126X,1HI,/126X,1HV,/121X,F8.2)
      WRITE(6,11) SPOT
   11 FORMAT(1X,130A1)
      WRITE(6,15)A
   15 FORMAT(60X,27HLINEAR EXPAND PLOT(RAD/SEC) ,34X,F8.2)
      RETURN
      END
C
C
      SUBROUTINE SAVER(ROOTR,ROOTI,ID1C,SAVE1,SAVE2,JZO,K1)
      DIMENSION SAVE1(100,100),SAVE2(100,100),ROOTR(100),ROOTI(100)
      IF(K1)30,9,10
    9 K1=1
   10 ID1=ID1C
      ID1C=ID1C+(K1-1)
      IF(ID1C.GE.100)GO TO 30
   50 DO 40 IZAP=K1,ID1C
      IZA=IZAP-(K1-1)
      SAVE1(JZO,IZAP)=ROOTR(IZA)
      SAVE2(JZO,IZAP)=ROOTI(IZA)
   40 CONTINUE
      K1=ID1C+1
      GO TO 20
   30 JZO=JZO+1
      ID1C=ID1
      GO TO 9
   20 RETURN
      END
C
C
C
C
      SUBROUTINE PLOTZR(SAVE1,SAVE2,ANUMB1,ANUMBZ,POINT,XA,XB,MI,PRNT,
     *STAR,NBB,NO)
      DIMENSION SAVE1(100,100),SAVE2(100,100),XA(2000),XB(2000)
      INTEGER POINT(130,100),DASH,BLANK,STAR
      LOGICAL PRNT
      IF(PRNT) GO TO 43
      DO 40 NAB=1,100
      DO 40 NBB=1,100
      IF(SAVE1(NAB,NBB))41,42,41
   42 IF(SAVE2(NAB,NBB))41,40,41
```

```
   41 ANUMB1=SAVE1(NAB,NBB)
      ANUMB2=SAVE2(NAB,NBB)
      ANUMB2=ABS(ANUMB2)
   43 AZZ=ANUMB2
      AZZ=AZZ*100000.0
      NZZ=AZZ
      IF(NZZ.EQ.0) GO TO 45
   50 NBB=NBB+1
   45 CALL EXCUTE(ANUMB1,ANUMB2,POINT,XA,XB,MI,NO,STAR)
      IF(PRNT)RETURN
   40 CONTINUE
   70 CALL WRITIT(XA,XB)
      CALL PREPAR(POINT)
      RETURN
      END
C
C
C
C
      SUBROUTINE EXCUTE(ANUMB1,ANUMB2,POINT,XA,XB,MI,NO,STAR)
      LOGICAL SKIP1,SKIP2
      LOGICAL LESS
      DIMENSION XA(2000),XB(2000)
      INTEGER POINT(130,100),DASH,BLANK,STAR
      DATA      K1/10/,K2/100/,K3/1000/,K4/10000/,NEGONE/-1/
      LESS=.FALSE.
      I=0
      J=0
      CALL MEMORY(ANUMB1,ANUMB2,NO,XA,XB)
      IF(ABS(ANUMB1).GT.10000.0) GO TO 50
      IF(ABS(ANUMB2).GT.10000.0) GO TO 50
      IF(ABS(ANUMB1).EQ.0.0) GO TO 500
      IF(ABS(ANUMB2).EQ.0.0) GO TO 500
      IF(ABS(ANUMB2).LT.0.001) GO TO 50
      IF(ABS(ANUMB1).LT.0.001) GO TO 50
  500 CALL SCALE1(K1,K2,K3,K4,NEGONE,I,ANUMB1,ICONS,LESS,SKIP1)
      CALL SCALE2(K1,K2,K3,K4,J,JCONS,ANUMB2,SKIP2)
      CALL WPOINT(J,JCONS,L,ANUMB2,SKIP2)
      CALL SPOINT(I,ANUMB1,LESS,ICONS,L,POINT,SKIP1,NO,XA,XB,STAR)
   50 RETURN
      END
C
C
C
C
      SUBROUTINE SCALE1(K1,K2,K3,K4,NEGONE,I,ANUMB1,ICONS,LESS,SKIP1)
      LOGICAL LESS
      LOGICAL SKIP1
      SKIP1=.FALSE.
      ICONS=1
      I=1
      AKEEP=ANUMB1
   21 NUMB1=ANUMB1
      NUMB=IABS(NUMB1)
      IF(NUMB.EQ.0)GOTO 12
      IF(NUMB1)31,40,40
   40 IF(NUMB1.GE.10)GOTO 11
      GO TO 50
   31 LESS=.TRUE.
      XXX=-NUMB1
```

```
      NUMB1=XXX
      GOTO 40
   12 GOTO(1,2,3,4),I
    1 ANUMB1=AKEEP
      ICONS=K1
      RK1=K1
      ANUMB1=ANUMB1*RK1
      I=2
      GOTO 21
    2 ANUMB1=AKEEP
      ICONS=K2
      RK2=K2
      ANUMB1=ANUMB1*RK2
      I=3
      GOTO 21
    3 ANUMB1=AKEEP
      ICONS=K3
      RK3=K3
      ANUMB1=ANUMB1*RK3
      I=4
      GOTO 21
    4 ANUMB1=AKEEP
      ICONS=K4
      RK4=K4
      ANUMB1=ANUMB1*RK4
      NUMB1=ANUMB1
      IF(NUMB1.EQ.0) GO TO 51
      GO TO 50
   11 SKIP1=.TRUE.
      GO TO (6,7,8,9),I
    6 ANUMB1=AKEEP
      ICONS=K1
      RK1=K1
      ANUMB1=ANUMB1/RK1
      I=2
      GOTO 21
    7 ANUMB1=AKEEP
      ICONS=K2
      RK2=K2
      ANUMB1=ANUMB1/RK2
      I=3
      GOTO 21
    8 ANUMB1=AKEEP
      ICONS=K3
      RK3=K3
      ANUMB1=ANUMB1/RK3
      I=4
      GOTO 21
    9 ANUMB1=AKEEP
      ICONS=K4
      RK4=K4
      ANUMB1=ANUMB1/RK4
      GO TO 50
   51 I=5
   50 RETURN
      END
C
C
C
C
```

```
      SUBROUTINE SCALE2(K1,K2,K3,K4,J,JCONS,ANUMB2,SKIP2)
      LOGICAL SKIP2
      SKIP2=.FALSE.
      JCONS=1
      J=1
      ANUMB2=ABS(ANUMB2)
      BKEEP=ANUMB2
   20 NUMB2=ANUMB2
      IF(NUMB2.EQ.0)GO TO 10
      IF(NUMB2.GE.10)GO TO 11
      GO TO 50
   10 GOTO(1,2,3,4),J
    1 ANUMB2=BKEEP
      JCONS=K1
      RK1=K1
      ANUMB2=ANUMB2*RK1
      J=2
      GO TO 20
    2 ANUMB2=BKEEP
      JCONS=K2
      RK2=K2
      ANUMB2=ANUMB2*RK2
      J=3
      GO TO 20
    3 ANUMB2=BKEEP
      JCONS=K3
      RK3=K3
      ANUMB2=ANUMB2*RK3
      J=4
      GO TO 20
    4 ANUMB2=BKEEP
      JCONS=K4
      RK4=K4
      ANUMB2=ANUMB2*RK4
      NUMB2=ANUMB2
      IF(NUMB2.EQ.0) GO TO 51
      GO TO 50
   11 SKIP2=.TRUE.
      GO TO (6,7,8,9),J
    6 JCONS=K1
      ANUMB2=BKEEP
      RK1=K1
      ANUMB2=ANUMB2/RK1
      J=2
      GO TO 20
    7 JCONS=K2
      ANUMB2=BKEEP
      RK2=K2
      ANUMB2=ANUMB2/RK2
      J=3
      GO TO 20
    8 JCONS=K3
      ANUMB2=BKEEP
      RK3=K3
      ANUMB2=ANUMB2/RK3
      J=4
      GO TO 20
    9 JCONS=K4
      ANUMB2=BKEEP
      RK4=K4
```

```
      ANUMB2=ANUMB2/RK4
      GO TO 50
   51 J=5
   50 RETURN
      END
C
C
C
C
      SUBROUTINE WPOINT(J,JCONS,L,ANUMB2,SKIP2)
      LOGICAL SKIP2
      L=0
      IDELTA=0
      IF(ANUMB2.GE.9.2) IDELTA=12
      IF(ANUMB2.GE.8.0) GO TO 200
      IF(ANUMB2.GE.6.9) GO TO 201
      IF(ANUMB2.GE.5.9) GO TO 202
      IF(ANUMB2.GE.5.0) GO TO 203
      IF(ANUMB2.GE.4.2) GO TO 204
      IF(ANUMB2.GE.3.5) GO TO 205
      IF(ANUMB2.GE.2.9) GO TO 206
      IF(ANUMB2.GE.2.4) GO TO 207
      IF(ANUMB2.GE.1.9) GO TO 208
      IF(ANUMB2.GE.1.5) GO TO 209
      IF(ANUMB2.GE.1.2) GO TO 210
      GO TO 211
  200 IF(ANUMB2.LT.9.2) IDELTA=11
  201 IF(ANUMB2.LT.8.0) IDELTA=10
  202 IF(ANUMB2.LT.6.9) IDELTA=9
  203 IF(ANUMB2.LT.5.9) IDELTA=8
  204 IF(ANUMB2.LT.5.0) IDELTA=7
  205 IF(ANUMB2.LT.4.2) IDELTA=6
  206 IF(ANUMB2.LT.3.5) IDELTA=5
  207 IF(ANUMB2.LT.2.9) IDELTA=4
  208 IF(ANUMB2.LT.2.4) IDELTA=3
  209 IF(ANUMB2.LT.1.9) IDELTA=2
  210 IF(ANUMB2.LT.1.5) IDELTA=1
  211 IF(ANUMB2.LT.1.2)IDELTA=0
      IDELTA=13-IDELTA
      IF(J.EQ.5) GO TO 50
      IF(SKIP2) GO TO 41
      IF(JCONS.EQ.1)GOTO 1
      IF(JCONS.EQ.10)GOTO 10
      IF(JCONS.EQ.100)GOTO 100
      IF(JCONS.EQ.1000)GOTO 1000
      IF(JCONS.EQ.10000)GOTO 10000
1     L=73+IDELTA
      GO TO 40
   10 L=86+IDELTA
      GO TO 40
  100 L=99+IDELTA
      GO TO 40
 1000 L=112+IDELTA
      GO TO 40
10000 L=125
      GO TO 40
   41 KCONS=JCONS+1
      SKIP2=.FALSE.
      IF(KCONS.EQ.11)GOTO 11
      IF(KCONS.EQ.101)GOTO 101
```

```
      IF(KCONS.EQ.1001)GOTO 1001
      IF(KCONS.EQ.10001)GOTO 10001
   11 L=60+IDELTA
      GO TO 40
  101 L=47+IDELTA
      GO TO 40
 1001 L=34+IDELTA
      GO TO 40
   50 L=125
      GO TO 40
10001 L=34
   40 RETURN
      END
C
C
C
C
      SUBROUTINE SPOINT(I,ANUMB1,LESS,ICONS,L,POINT,SKIP1,NO,XA,XB,STAR)
      DIMENSION XA(2000),XB(2000)
      INTEGER POINT(130,100),DASH,BLANK,STAR,Q
      INTEGER STAR1,STAR2
      DATA STAR1/1HO/,STAR2/1H*/
      DATA BLANK/1H /,DASH/1H-/,          Q/1HI/
      LOGICAL SKIP1         ,LESS
      AKEEP=ANUMB1
      ANUMB1=ABS(AKEEP)
      KOOL=0
      IF(I.EQ.5)GO TO 1000
      IF(ANUMB1.GE.7.1)KOOL=7
      IF(ANUMB1.GE.5.0)GO TO 1011
      IF(ANUMB1.GE.4.0)GO TO 1111
      IF(ANUMB1.GE.3.1)GO TO 1211
      IF(ANUMB1.GE.2.3)GO TO 1311
      IF(ANUMB1.GE.1.6)GO TO 1411
      IF(ANUMB1.GE.1.0)GO TO 1511
      GO TO 4011
1011  IF(ANUMB1.LT.7.1)KOOL=6
1111  IF(ANUMB1.LT.5.0)KOOL=5
1211  IF(ANUMB1.LT.4.0)KOOL=4
1311  IF(ANUMB1.LT.3.1)KOOL=3
1411  IF(ANUMB1.LT.2.3)KOOL=2
1511  IF(ANUMB1.LT.1.6)KOOL=1
 4011 NCRMNT=8-KOOL
      IF(LESS)GO TO 40
      IF(SKIP1)GO TO 41
      IF(ICONS.EQ.1)GOTO 1
      IF(ICONS.EQ.10)GOTO 10
      IF(ICONS.EQ.100)GOTO 100
      IF(ICONS.EQ.1000)GOTO 1000
    1 KOOL=71+KOOL
      GO TO 50
   10 KOOL=64+KOOL
      GO TO 50
  100 KOOL=57+KOOL
      GO TO 50
 1000 KOOL=50+KOOL
      GO TO 50
   41 LCONS=ICONS+1
      SKIP1=.FALSE.
      IF(LCONS.EQ.11)GOTO 11
```

```
      IF(LCONS.EQ.101)GOTO 101
      IF(LCONS.EQ.1001)GOTO 1001
   11 KOOL=78+KOOL
      GO TO 50
  101 KOOL=85+KOOL
      GO TO 50
 1001 KOOL=92+KOOL
   50 LESS=.FALSE.
      IF(POINT(L,KOOL).EQ.DASH) GO TO 51
      IF(POINT(L,KOOL).EQ.0    ) GO TO 51
   14 IF(POINT(L,KOOL).EQ.STAR1) GO TO 15
      IF(POINT(L,KOOL).EQ.STAR2) GO TO 15
      IF(STAR.EQ.STAR1) GO TO 51
      IF(STAR.EQ.STAR2) GO TO 51
      IF(POINT(L,KOOL).NE.BLANK) GO TO 70
   51 POINT(L,KOOL)=STAR
      ANUMB1=AKEEP
      GO TO 70
   15 L=L+1
      IF(L.EQ.130) GO TO 51
      GO TO 14
   40 IF(SKIP1) GO TO 42
      IZ=ICONS+2
      IF(IZ.EQ.3)GOTO 3
      IF(IZ.EQ.12)GOTO 12
      IF(IZ.EQ.102)GOTO 102
      IF(IZ.EQ.1002)GOTO 1002
    3 KOOL=21+NCRMNT
      GO TO 50
   12 KOOL=28+NCRMNT
      GO TO 50
  102 KOOL=35+NCRMNT
      GO TO 50
 1002 KOOL=42+NCRMNT
      GO TO 50
   42 IY=ICONS+3
      SKIP1=.FALSE.
      IF(IY.EQ.13)GOTO 13
      IF(IY.EQ.103)GOTO 103
      IF(IY.EQ.1003)GOTO 1003
      IF(IY.EQ.10003)GOTO 10003
   13 KOOL=14+NCRMNT
      GO TO 50
  103 KOOL=7+NCRMNT
      GO TO 50
 1003 KOOL=NCRMNT
      GO TO 50
10003 POINT(L,1)=STAR
      GO TO 70
   70 RETURN
      END
C



      SUBROUTINE PREPAR(POINT)
      INTEGER POINT(130,100),PRT
      DATA PRT /6/
      WRITE(6,1)
    1 FORMAT(1H1)
```

```
      WRITE(6,11)
   11 FORMAT(///)
      WRITE(PRT,14)
   14 FORMAT(75X,8HLOG PLOT,/58X,42HCOMPLEX FREQUENCY PLANE,LEFT HAND QU
     *ADRANT,/75X,9H(RAD/SEC)   )
      WRITE(6,12)
   12 FORMAT(32X,5H10000,9X,4H1000,9X,3H100,10X,2H10,12X,1H1,11X,2H.1,10
     *X,14H.01<---J-OMEGA,/ 18X,11HMINUS SIGMA)
   13 FORMAT(32X,5H10000,9X,4H1000,9X,3H100,10X,2H10,12X,1H1,11X,2H.1,10
     *X,14H.01<---J-OMEGA,/ 18X,11H PLUS SIGMA)
      DO 50 I=1,130
   50 POINT(I,50)=POINT(I,51)
      WRITE(PRT,10)POINT
   10 FORMAT(1X,130A1)
      WRITE(6,13)
      WRITE(PRT,15)
   15 FORMAT(75X,8HLOG PLOT,/58X,43HCOMPLEX FREQUENCY PLANE,RIGHT HAND Q
     *UADRANT,/75X,9H(RAD/SEC)   )
      RETURN
      END
C
C
C
C
      SUBROUTINE MEMORY(ANUMB1,ANUMB2,NO,XA,XB)
      DIMENSION XA(2000),XB(2000)
      XA(NO)=ANUMB1
      XB(NO)=ABS(ANUMB2)
      NO=NO+1
      RETURN
      END
C
C
C
C
      SUBROUTINE WRITIT(XA,XB)
      DIMENSION XA(2000),XB(2000)
      WRITE(6,1)
    1       FORMAT(1H1)
      WRITE(6,11)
   11 FORMAT(5X,48HTHE FOLLOWING  ROOTS ARE PLOTTED ON THE LOG PLOT,/
     15X,99HROOTS AT THE ORIGIN ARE NOT PRINTED OR PLOTTED, ROOTS ON THE
     * J-OMEGA AXIS ARE NOT PLOTTED.                                   ,
     2,//16X,5HSIGMA,25X,7HJ-OMEGA,//)
      DO 77 IZO=1,2000
      XC=XB(IZO)
      IF(XC.GT.0.0) XC=-XC
      IF(XC.NE.0.0) GO TO 22
   20 IF(XA(IZO))22,40,22
   40 K1=IZO
      K2=IZO+12
      DO 50 K=K1,K2
      IF(XA(K).NE.0.0) GO TO 77
   50 IF(XB(K).NE.0.0) GO TO 77
      GO TO 60
   22 WRITE(6,10)XA(IZO),XC
   10 FORMAT(5X,F20.9,10X,F20.9)
   77 CONTINUE
   60 RETURN
      END
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13. ABSTRACT

An existing program that determines the root locus of n'th order polynominals has been modified to provide plots of these loci in the complex frequency plane using a standard line printer. A methodology that combines the computational capabilities of this root locus program with a variable scale graphical display of selectable regions of the complex frequency plane is presented. A listing of the Fortran IV source deck of the modified program and two examples are included.

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